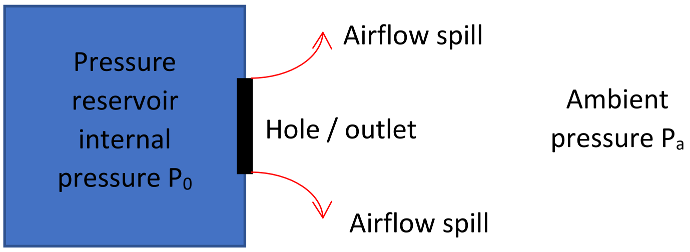
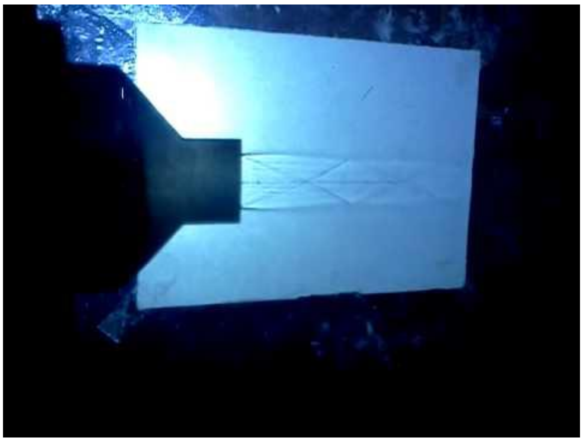
Four options for supersonic windtunnels

1. A simple hole.

This option is shown diagrammatically in the diagram below:



And here's a picture of something similar (with converging section before hole):



The flow is sonic at the hole when the flow is choked at a pressure difference between reservoir and ambient is:

$$\frac{P_1}{P_0} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}}$$

For standard air this means:

$$P_1 \leq 0.528 P_0$$

When in this condition the mass flow rate can only be increased by increasing the reservoir pressure (and is not affected by the ambient pressure):

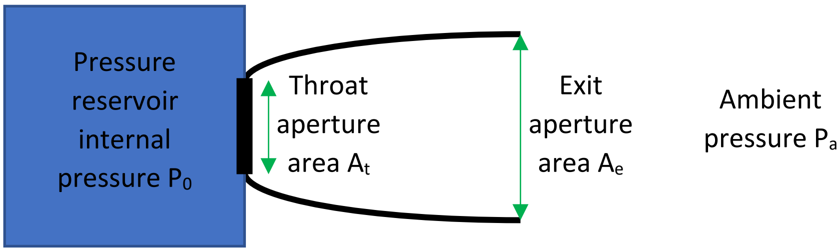
$$\dot{m} = C_d A \sqrt{\gamma \rho_0 P_0 \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

Where C_d is the discharge coefficient (basically depends on the shape and type of hole) and A is the hole area.

When in this situation the flow spills out over the edges of the hole and the speed increases somewhat (because the fluid is supersonically expanding) but in an unreliable way. The velocity then decreases through the series of shockwaves and expansion-fans of the shock diamonds in the hole's wake as the pressure of the stream equalises with ambient.

2. Simple diverging duct

This situation is illustrated in the diagram below:



An explanation and example of this is given in Anderson "Fundamentals of Aerodynamics" (Section 10.5, p 579 in the "Compressible flow through Nozzles" chapter of the 3rd edition).

There are to parameters which need to be found in order to design this variant. The first is the ratio of throat (hole) area A_t to exit aperture area A_e (A_e/A_t). The second is the required pressure ratio to achieve the flow you want (P_0/P_a). We must then manufacture a nozzle with the correct size ratios and connect this to a cylinder which can achieve the correct pressure (and have enough capacity to sustain it for a period of time).

The required ratios can be obtained from appendix A "isentropic flow properties" of the book mentioned above or from the isentropic equations:

$$\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

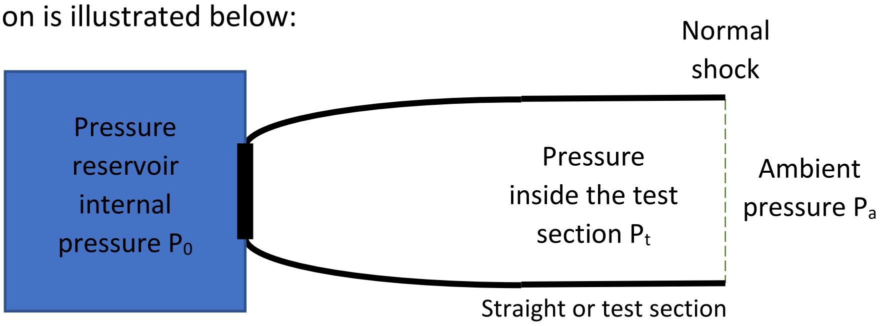
$$\frac{A}{A_t} = \frac{1}{M}\left(\frac{1 + \frac{\gamma - 1}{2} M^2}{\frac{\gamma + 1}{2}}\right)^{\frac{\gamma + 1}{2(\gamma - 1)}}$$

An example is given in Anderson's text for M = 2.5 at the outlet.

The problem with this solution is the high-pressure ratio required (which means a sustained high-pressure air source – expensive and large).

3. Diverging duct and straight (or test) section

This situation is illustrated below:



In this configuration a normal shock exists at the exit of the nozzle as shown. This is the boundary between supersonic and subsonic flow and also acts as a diffuser to return the flow to normal pressure (although not straight away).

We can design this configuration by first looking up normal shock tables for the Mach number we want in the test section (for example appendix B in Anderson), or using the normal shock equations:

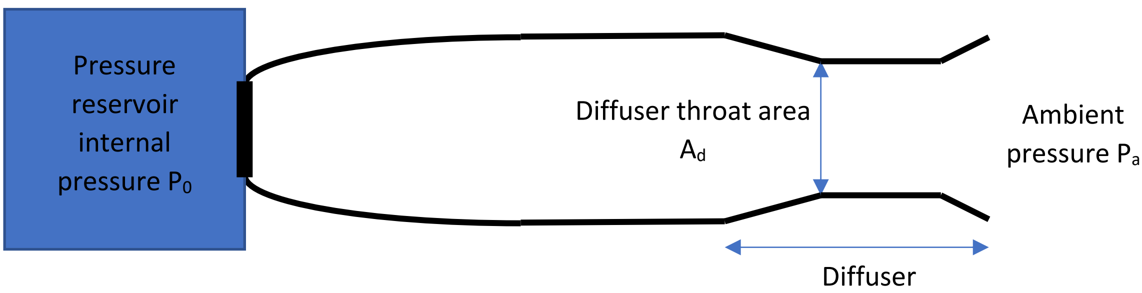
$$\frac{p_1}{p_0} = \frac{2\gamma M_0^2 - (\gamma - 1)}{\gamma + 1}$$

This gives us P_t inside the duct. This pressure is established from the hole and divergent section exactly as shown in the previous example (2). However, because P_t is much lower than the ambient pressure (due to the normal shock) the required reservoir pressure is now much smaller. Again, Anderson gives an example of this for M = 2.5 in his text.

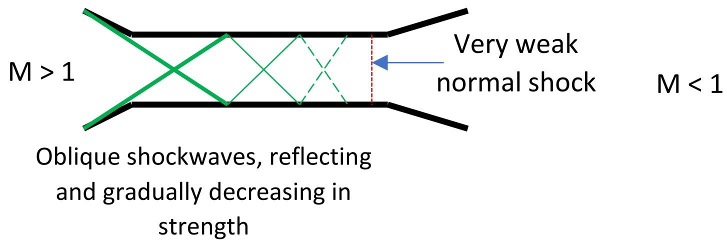
However, this design suffers from a problem – it is very difficult to establish the normal shockwave stably at the end of the tunnel – it tends to wonder in and out of the duct causing uncertainty in the resulting flow. The final design below allows this to be overcome.

4. Diverging duct, straight section and diffuser

This situation is shown below:



A diffuser is a structure which reduces the velocity of the flow (in this case to subsonic). In the previous example this was done by the normal shockwave at the end of the text section. However, as noted this approach has some practical problems. We can produce a very similar effect by using a series of oblique shockwaves (and then a very weak normal shock as the flow finally becomes subsonic) as shown in the diagram below. This produces a similar effect to the strong normal shockwave but has fewer of the practical problems associated with it and can also be more efficient (although not always):



As the oblique shockwaves travel down the duct, slowing the flow, they gradually diminish in strength and terminate in a supersonic to subsonic transition (which is always heralded by a normal shock – though much weaker this time).

This type of structure is what is used in large-scale professional supersonic windtunnels.

It can be shown that the diffuser throat area (A_d) needs to be larger than the reservoir throat area (A_t) for this configuration to work correctly. In fact, the minimum ratio of throat areas is given by the ratio of total pressures between the sections of the system:

$$\frac{A_d}{A_t} = \frac{P_{1(total)}}{P_{2(total)}}$$

So how do we approach this design? Well we can start by assuming that the total pressure ratio required is the same as for the normal shockwave in the previous example. We can get this ratio by looking it up in Appendix B of Anderson. Or by using the total pressure equation:

$$\frac{p_{t1}}{p_{t0}} = \left[\frac{(\gamma+1)M_0^2}{(\gamma-1)M_0^2+2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{(\gamma+1)}{2\gamma M_0^2-(\gamma-1)}\right]^{\frac{1}{\gamma-1}}$$

(the required value for area ratio is actually the inverse of this).

There are no "hard and fast" rules to complete the design as the diffuser itself can take many different forms in terms of shape and length (a combination of experimentation and simulation should be used). An example of this is given calculation is given in Anderson for Mach 2 (example 10.4 in the 3rd edition).