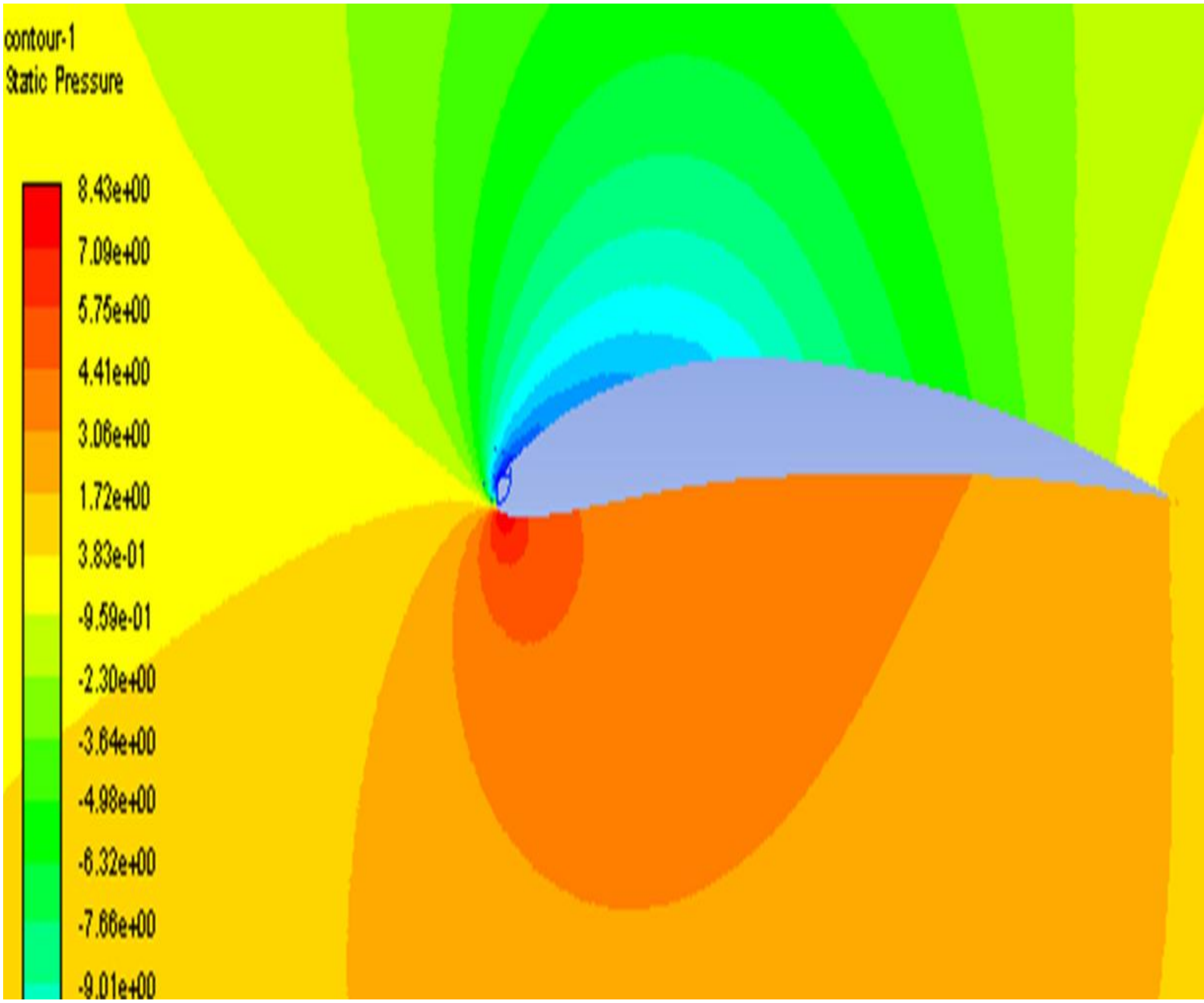


FLUIDS 4

AEROFOILS

In this section we will cover the parameters and theory of aerofoils; learn to do basic calculations involving them and find out about the more advanced methods used in their design.

FLUIDS 4

AEROFOILS

OVERVIEW

In this section you will learn about:

- The theories of lift and drag production on an aerofoil
- How the dimensions and physical parameters of an aerofoil are specified
- Types and causes of drag
- How to do calculations involving lift, drag and pitching moment
- How to use lift and drag graphs
- The nature of the stall
- Three dimensional effects on aerofoils
- Theoretical methods of parameter calculation and design
- The use of CFD in aerofoil design
- Supersonic and transonic aerofoils

ASSUMED KNOWLEDGE

In this subject it is assumed that you already have knowledge about the following topics:

- *A good knowledge of fluid parameters*
- *Incompressible flow including Bernoulli, Continuity, Reynold's Number and Thrust equations*
- *Isentropic compressible flow (qualitatively)*
- *The basic (qualitative) ideas behind normal and oblique shockwaves*
- *The basic ideas behind CFD and some practice of its use*

OBJECTIVE

To understand aerofoil terminology, parameters and design methods.

Aerofoils (or airfoils) are shapes which, when a fluid flows around them, provide a useful force in a particular direction. The most common conception of an aerofoil is an aircraft wing. However, there are many other examples, including:

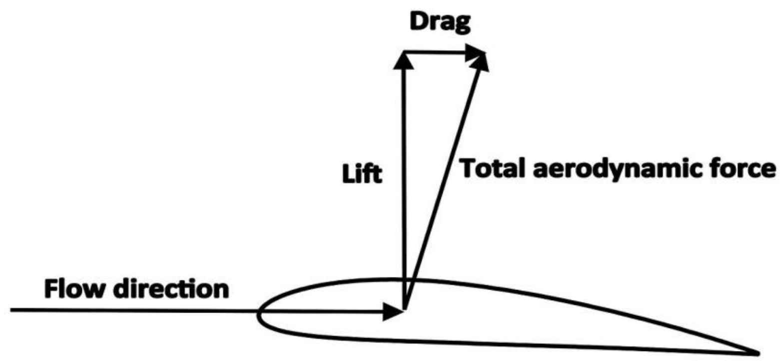
- The blades of a wind or tidal turbine. Here the generated force causes the blades to turn, generating a torque which is turned into electrical power by a generator.
- Similarly, the blades of a steam or gas turbine. Here the blades are typically small because the pressures and resulting fluid energy is much greater. However, the principle is the same and the additive force on all the blades turns the central spool which produces power.
- The sail of a sailing ship. Although it might not seems like this at first sight, the sail, when at an angle to the airflow, acts like an aerofoil, which generates thrust for the vessel.

It is also, of course, possible to use the reverse process to make useful machines - to supply mechanical power to an aerofoil, so as to effect the flow of the fluid. Many pumps and compressors operate in this way in order to change fluid velocity, direction or pressure.

TOPIC 1 - THE INTUITIVE THEORY OF LIFT ON A WING

To show how forces are generated on an aerofoil, let's consider an aeroplane wing as an example. A wing sitting in an airflow generates an upwards force called *lift*. It is this lift which keeps the aircraft aloft (and is also the motive force in, for example, turbines). The flow also generates a *drag* force retarding the aerofoil, colloquially known as air-resistance - but more detail on this later.

The diagram overleaf shows the relationship between lift, drag and the total resultant force on a wing.



(Creative Commons image “airfoil lift and drag” by J Doug McLean, license: CC BY-SA 3.0)

If this aerofoil was the wing of an aircraft in level flight at constant speed, the lift would exactly balance the weight of the plane and the thrust from the engines would balance the drag.

Figure 1, Lift, drag and total force on an aerofoil

It may surprise you to know that, despite aerofoils being simple shapes, very common, and having been perfected a century ago; there is still some argument and controversy, in scientific circles, over how they work. Two rival explanations of lift are commonly discussed - however, most engineers think that these are actually equivalent to each other and amount to the same thing, looked at from different perspectives.

Explanation 1 - Pressure distribution

This explanation uses the pressure distribution across the aerofoil’s surface to explain the forces on it. In particular, the top side has a lower pressure than the bottom causing the upwards force of lift.

When we put a smoothly-curved surface into a fluid stream, the fluid, due to its viscosity, tends to follow the surface. This phenomenon is called the *Coanda Effect*. You can see it in action by putting a smooth item like the back of a spoon or the round-body of a pen into a laminar water-stream from a tap as shown overleaf.

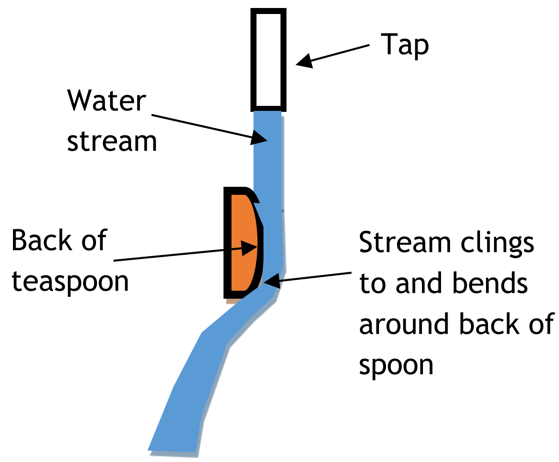


Figure 2, The Coanda effect

For the effect to work, the surface must have a gentle curve - and this is the main reason why low-speed aerofoils have their shape.

To accommodate this bending, the flow streamlines must bunch closer together as shown below.

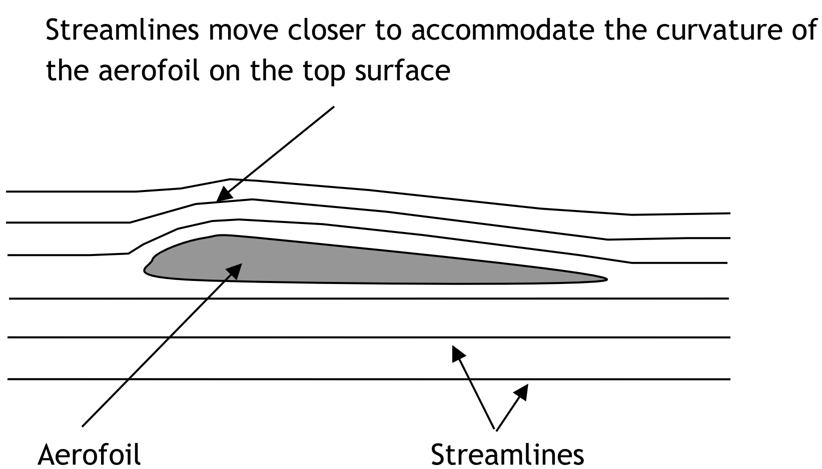


Figure 3, Stream-lines around an aerofoil

You might also consider the similar effect as an internal flow encounters a restriction in pipe width as shown overleaf.

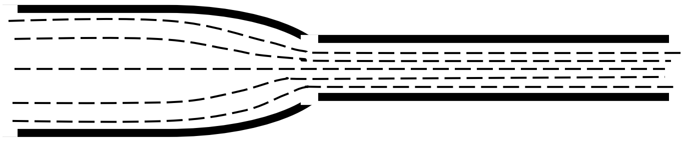


Figure 4, Flow through a gently narrowing pipe

Removing the upper part of the pipe, results in the situation shown – note the similarity to the streamlines flowing over the top of the aerofoil in figure 3.

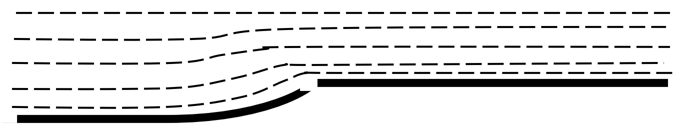
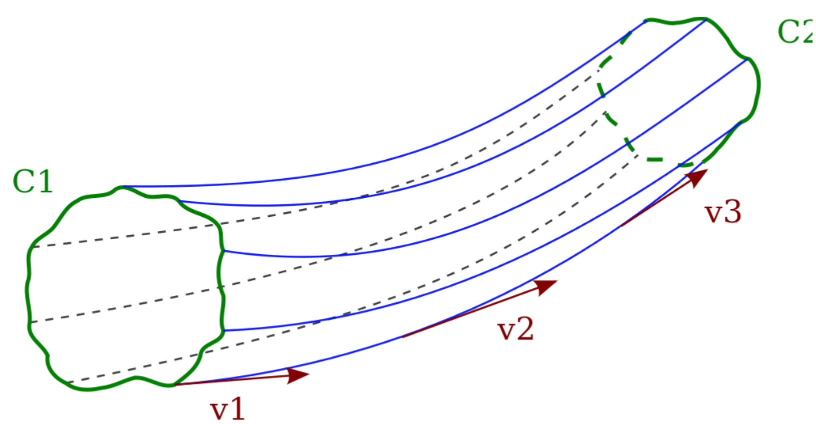


Figure 5, Upper part of pipe removed

No fluid travels across stream-lines (as you can see from their uninterrupted smoke-trails in a wind tunnel). So, in three dimensions they form a structure are rather like a pipe (in fact called a *stream-tube*), as shown below. You can perhaps see from this, that bunching of the lines implies faster flow - because, like a pipe, the flow-rate along the tube is constant – it is subject to the continuity equation ($AV = constant$ for incompressible flow).



(Image “Streamlines and streamtubes” by Twisp, license: listed as “copyright free, public domain” by author)

Figure 6, A streamtube constructed from streamlines

But we know from Bernoulli's Equation that (assuming, incompressible, invisid flow and that the height-change is negligible):

$$p_1 + \rho \frac{v_1^2}{2} = p_2 + \rho \frac{v_2^2}{2}$$

So as the velocity over the aerofoil is faster (as shown by the bunched streamlines) $v_2 > v_1$ and so, because density is constant, $p_2 < p_1$ and therefore the static pressure is less on the top surface of the wing. Further, since $F = pA$ the higher pressure on the lower side results in an upwards force - lift.

Before leaving this topic, it is worth noting that the total resultant force (and therefore both lift and drag) are a result of the pressure distribution around the aerofoil as shown (the yellow dot on the aerofoil, through which the resultant force can be thought of as acting, is called the *centre of pressure*).

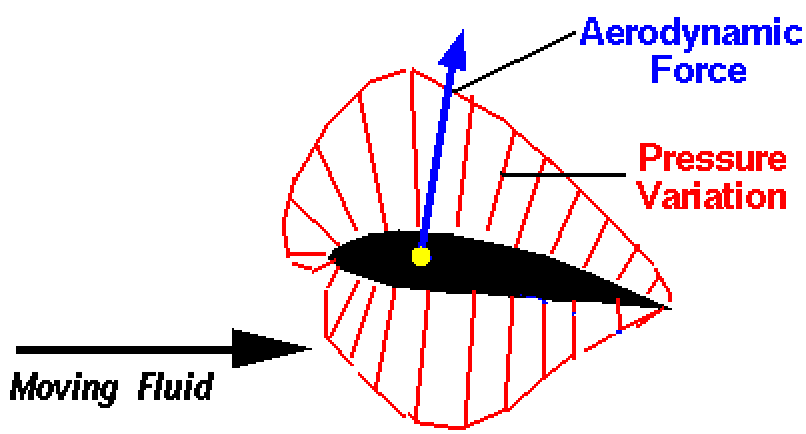


Figure 7, Pressure distribution around aerofoil and resultant aerodynamic force

Explanation 2 - Change of momentum

Another explanation for lift is, that it is the reaction force to the diversion of air downwards by the aerofoil, as shown below.

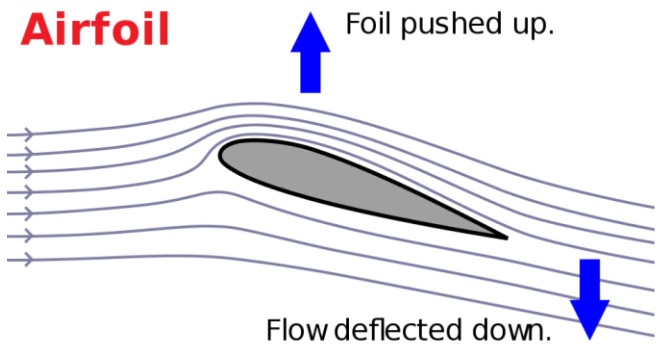


Figure 8, The downwards deflection of air by an aerofoil results in an upwards reaction (lift)

The lift force is the "equal and opposite reaction force" which results from the change of momentum in the fluid stream:

$$F = \frac{d(mv)}{dt}$$

Incorrect Explanations

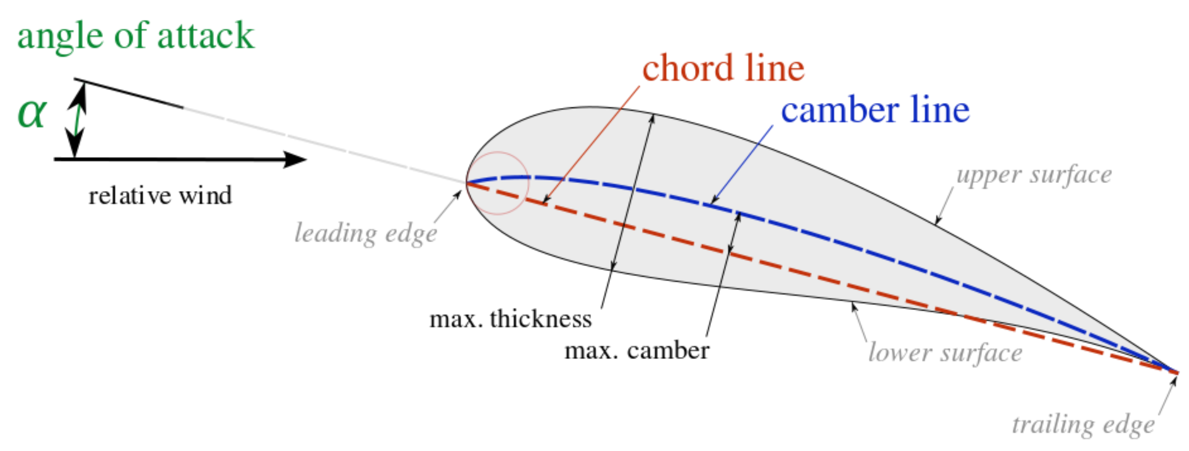
There are also a number of explanations for the origin of lift, which are accepted as false. I won't go into these here - but you can find out about them by completing the task below.

TASK 1

Read the section on incorrect explanations on the Wikipedia page on "Lift".

TOPIC 2 - AEROFOIL DIMENSIONS AND PARAMETERS

The diagram below shows the important dimensions of an aerofoil.



(Creative Commons image "wing profile nomenclature" by Olivier Cleynen, license: CC0)

Figure 9, Aerofoil physical parameters

The *chord-line* is that straight line joining the leading and trailing edges of the aerofoil. The *chord length* is the length of this (straight) line. The mean *camber-line* is a line drawn which is exactly halfway between the top and bottom surfaces of the aerofoil. For the purposes of aerofoil calculations a mathematical function $Z(x)$ is often synthesized to describe its curve. An aerofoil is completely defined by this equation and a thickness function $T(x)$ which gives the thickness of the aerofoil at any point.

Two important types of types of aerofoil are the symmetrical and asymmetrical as shown in the diagram overleaf.

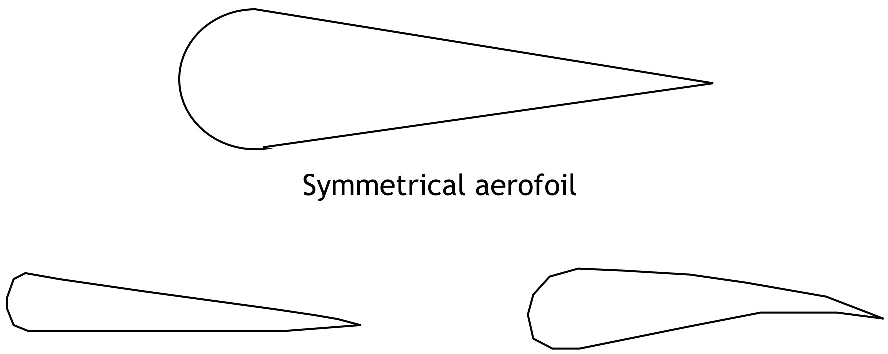


Figure 10, Symmetrical and asymmetrical aerofoils

TASK 2

Write down a function $Z(x)$ for the camber-line of a symmetrical aerofoil assuming it is 20 cm thick at its thickest and has a 1 m chord. Use SI units.

In general, symmetrical types allow aircraft to fly with more maneuverability (for example up-side down), but display less lift and require to be presented to the flow at an angle.

Talking of angle - this angle of presentation to the flow (called the angle of attack) is critical to the aerofoil's behavior, it is labelled α and is shown in figure 9. To be more precise, α is the angle between the flow presented to the aerofoil (called the *relative flow or wind*) and the chord line.

The geometrical relationship between this angle and the lift, drag and total resultant force is shown in the diagram overleaf.

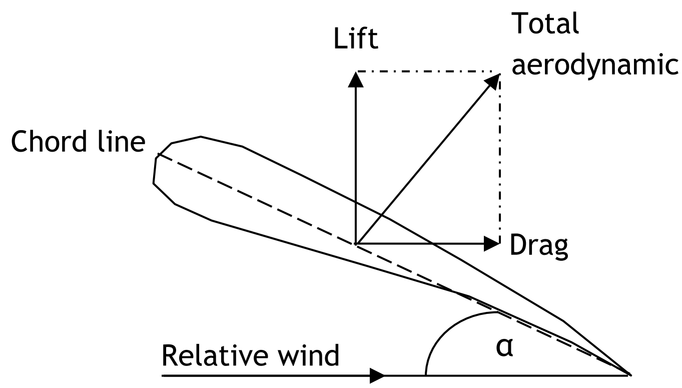


Figure 11, Diagram of forces and attack

We can see from this diagram, that the lift is the force measured at right angles to the relative wind (the direction of wind as the aerofoil sees it), while the drag is parallel to it.

TASK 3

If the measured total resultant aerodynamic force on an aerofoil is 10 kN at angle of 80^0 to the chord-line and the aerofoil's angle of attack is 15^o calculate the lift and drag components acting on the aerofoil. At what angle to the relative wind is the total force acting? Hint: draw a diagram of the situation first and use this to work out the angles.

Finally, the weight of the aerofoil (like the weight of any complex object) can be thought of as passing through a particular point (the system's balance-point) called the centre of gravity (CoG).

In a similar way, the aerodynamic force can also be thought of like this - its point of action is called the centre of pressure (CoP), as shown overleaf.

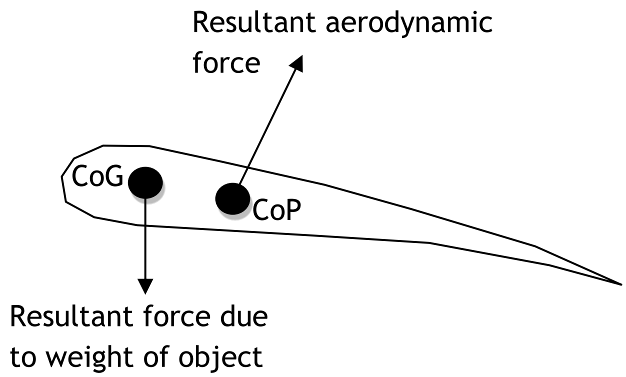


Figure 12, Centre of Mass and Centre of Pressure

These points, if they are not aligned (and they usually aren't), generate a turning moment on the aerofoil (which in the case of an aircraft in level flight, would be called a *pitching moment*). In an aircraft, this misalignment is counteracted by the moment produced by the horizontal tail – which, in a conventional aircraft configuration, produces a downwards force.

TASK 4

If the mass of the aerofoil discussed in task 3 is 50 kg and the structure is in the same configuration as before (task 3), calculate the pitching moment about the chord-line. The centre of pressure and centre of mass are 30 cm apart and the centre of gravity is ahead of the centre of pressure (it is closer to the leading edge). You can assume that both centres lie on the chord line and that the relative wind is horizontal. Hint: you need to consider the angles at which the forces are acting here (and the definition of a turning moment) – again a diagram would help.

Assuming that this is the wing of an aircraft and forgetting about the action of the tail for a moment – would this configuration of CoP and CoG be stable (would it return the aircraft to level flight if it pitched unexpectedly upwards)? Does this tell us anything about the configuration of a wing? Explain your answer.

TOPIC 3 – DRAG

The drag force retards the aerofoil - and is therefore generally a bad thing (unless it's a parachute or an airbrake!). It is the force which any object feels in the face of an airflow - also called air resistance. Just as the difference in pressure between the upper and lower surface produce lift the difference in horizontal pressures between the front and the back produces drag.

There are three principal sources of drag on a conventional, low-speed unconnected aerofoil. These are:

1. **Form drag.**

Form (or pressure) drag is why a flat plate or other similar object has more air-resistance than a streamlined shape. The plate disrupts the smooth passage of air and causes turbulence behind itself as shown below. This causes an unfavorable pressure distribution (higher pressure in front, lower at the back) which impedes the progress of the aerofoil.

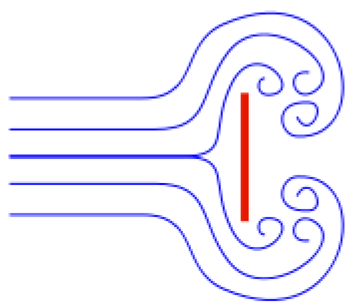


Figure 13, Flow around a plate, showing disruption – the origin of form drag

2. **Skin friction drag.**

This is caused by the friction (viscosity) of the air molecules rubbing against the aerofoil surface (in other words by the boundary-layer). It can be reduced by keeping all the surfaces smooth and polished.

Boundary layers can be either laminar or turbulent (depending on their physical characteristics and Reynold's Number). Laminar boundary layers display less drag - and some special devices (which you can do some research into yourself) have been devised to keep the layer as laminar as possible.

If the aerofoil meets another surface, which is not continuous with it (like the fuselage of the plane), the airflows around these two bodies interfere - creating another type of drag - *interference drag*.

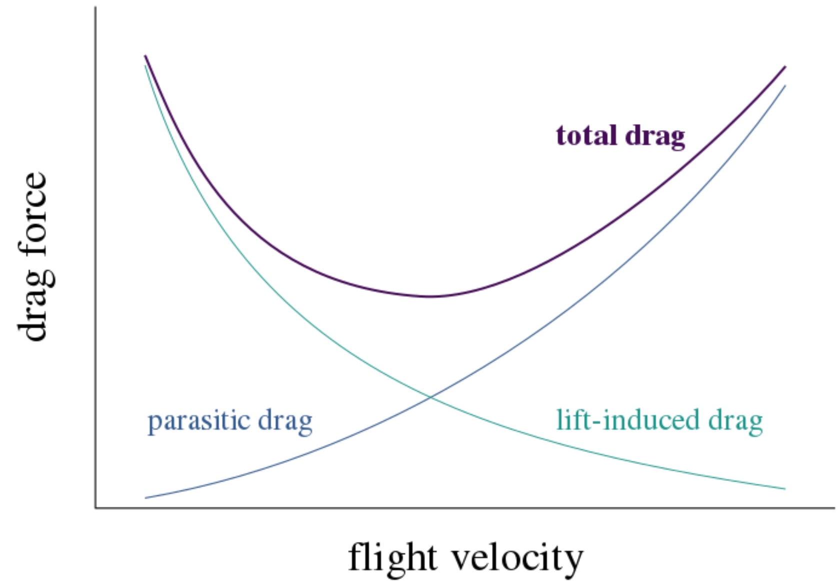
The addition of form, skin and interference drag is often referred to as "*Parasitic*" drag.

3. **Induced drag.**

When an aerofoil generates lift it creates a trail of vortices behind the wing tips - this is lost energy and results in induced drag. Induced drag is a direct result of lift - and is impossible to avoid in a conventional wing. We'll discuss it more in the section on 3D aerofoils.

It should be noted that supersonic aerofoils also suffer from another type of drag called wave-drag, which we'll also discuss later.

As the aerofoil speed increases, parasitic drag goes up (as the speed squared), but induced drag decreases (again with squared speed), as shown overleaf. This means there is a particular speed at which the drag is minimal, the lift to drag ratio is at its highest, and the aerofoil is at its most efficient.



(Creative Commons image “Drag curves for aircraft in flight” by Olivier Cleynen, license: CC0)

Figure 14, Total drag on a low-speed aerofoil

TOPIC 4 – BASIC CALCULATIONS USING AEROFOIL PARAMETERS

Since $F = p \times A$, the lift force on the aerofoil depends on the excess pressure (p) on its surface times its effective area (A). The pressure (as we will see) is difficult to calculate from first principles, but obviously depends on the density of air (ρ) around the aerofoil and its velocity (v) - which we can conveniently use the dynamic pressure to represent:

$$p_d = \rho \frac{v^2}{2}$$

However, it also depends on the shape of the aerofoil (which controls the pattern of airflow around it) and its angle of attack. Again this can be complex - consider all the different types and lengths of camber and thickness function possible. Fortunately, we can introduce a convenient constant to take its topology into account, called the *lift coefficient* (C_L).

The formula for lift then becomes:

$$L = p_d C_L A$$

Since we can easily measure the lift experimentally (using a force balance) and also A and p_d we can find the lift coefficient by measurement:

$$C_L = \frac{L}{p_d A}$$

A coefficient of drag can be similarly stated (D is the drag force, as in figures 1 and 11):

$$C_D = \frac{D}{p_d A}$$

As can a coefficient of turning (pitching) moment (torque) M. In this case c is the chord length:

$$C_M = \frac{M}{p_d Ac}$$

TASK 5

Consider the aerofoil discussed in the previous tasks. If the relative wind velocity is 200 kph at sea-level, the wing is square in plan with chord length 1 m and its area is 10 m^2. Calculate all three of the coefficients above.

What are the units of the three coefficients?

Another two measures which are often used to specify aerofoils are the *Lift to Drag Ratio* (note the minimum point of this on figure 14). In fact the lift-to-drag ratio of an aircraft can be shown to equal one over the tangent of its glide angle ϕ (the angle of decent of the aircraft when gliding, relative to the ground – Google "glide angle", if this is unclear):

$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{1}{tan\phi}$$

The maximum L/D ratio of an aerofoil also corresponds to the minimum drag shown in figure 14.

And finally the wing loading. This is the ratio of lift to area in any situation and the ratio of weight (W) to area in an aircraft in level flight:

$$\frac{L}{A} = \frac{W}{A} \; (in\ level\ flight)$$

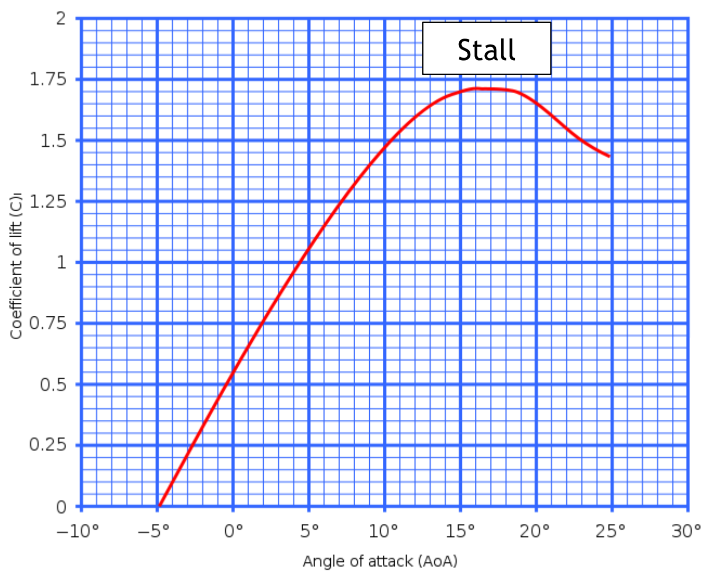
TASK 6

What is the glide angle of the wing above (assuming there is no effect from a fuselage), what is its loading in level flight?

What are the units of these two parameters?

TOPIC 5 – LIFT AND DRAG GRAPHS

As you might imagine, the lift coefficient depends strongly on α. The two can be plotted on a graph as shown.



(Creative Commons image "liftcurve" by Mysid, license: CC0)

Figure 15, Graph of lift-coefficient (C_L on the y axis) against angle of attack (α on the x axis)

The region labelled "stall" will be discussed in more detail in the next topic. Note however that at the stall the aerofoil suddenly loses lift.

You can see that lift increases roughly linearly up to the stall. The difference between symmetrical and non-symmetrical aerofoils is shown overleaf. Note how in the symmetric case there is no lift at $\alpha = 0$; the cambered aerofoils have higher lift, but stall at a lower angle.

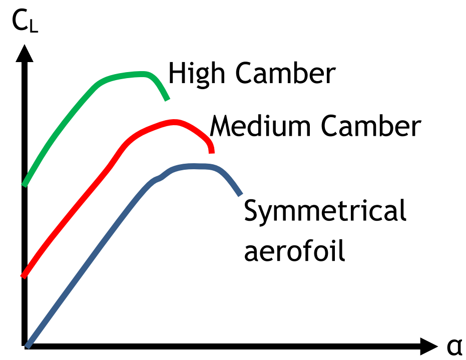


Figure 16, Lift graphs for symmetrical and cambered aerofoils

We can also plot a similar graph for drag - and this is often drawn on the same graph as lift as shown.

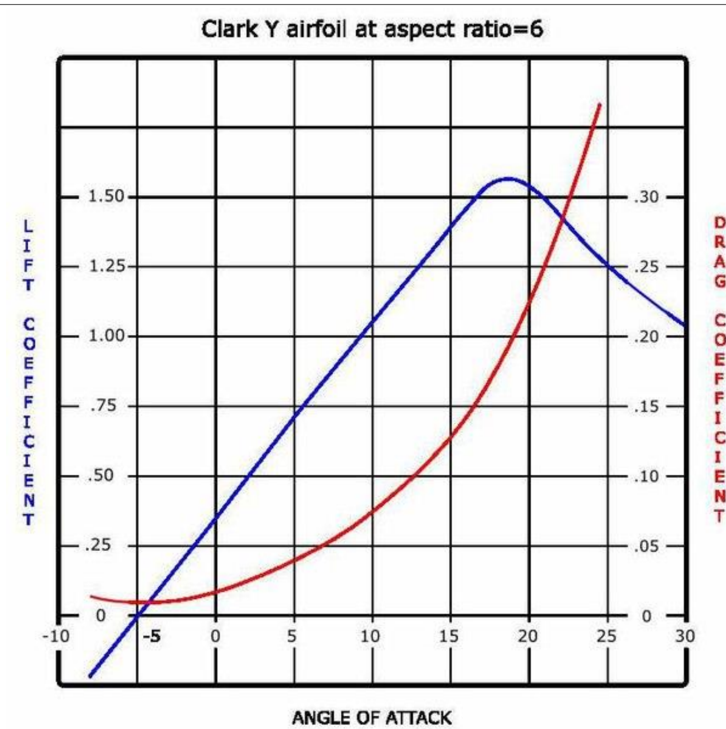


Figure 17, Combined lift and drag graph (drag line in red, D_L on y axis right-hand side)

Such graphs have been plotted experimentally for a large number of aerofoil shapes. These provide one the main means of selecting suitable aerofoils.

TASK 7

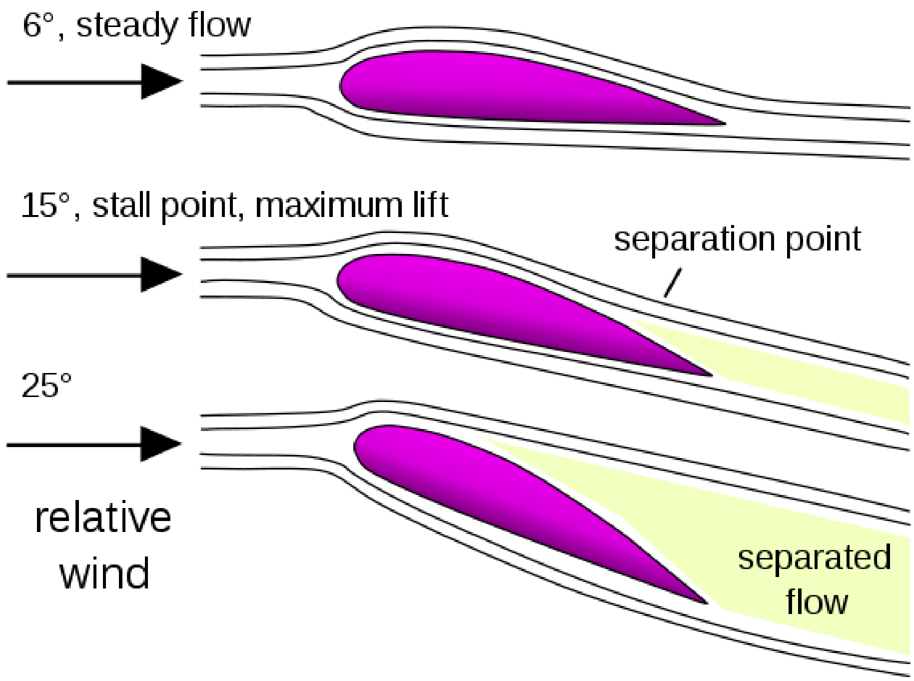
From the graph shown in figure 17, assuming a flow velocity of 200 kph at sea level and a wing area of 10 m^2, calculate the lift and drag force at $\alpha = 10^0$. What would be the glide angle if this were used as the wing of an aircraft?

TOPIC 6 – THE STALL

As mentioned above, at the point of stall, the lift force suddenly (or relatively suddenly) falls (often dramatically). In an aircraft, a stall would happen if the plane adopted too much of a nose-up attitude or slowed down too much. A pilot can recover from this, provided s/he has enough altitude, by pointing the nose down and allowing the aircraft to gather speed. However, you should note that all aerofoils can experience stall – including those you might not associate with it - (for example) gas turbine blades.

The stall occurs when the angle of attack becomes too great for the flow to follow the curve of the aerofoil all the way around. When this happens, the flow *separates* from the aerofoil as shown – the aerofoil has stalled.

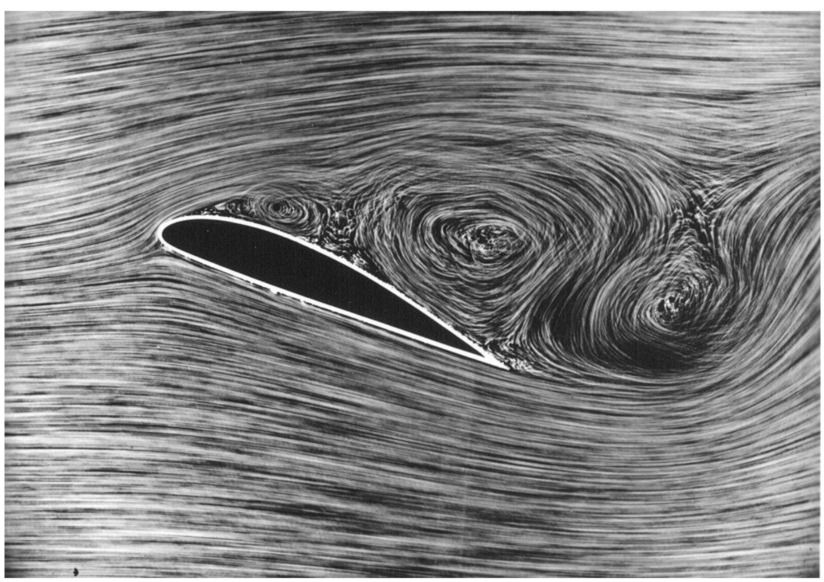
This situation is shown in the diagram overleaf.



(Image "Stallformation" by US airforce, license: listed as "copyright free, public domain" by authors)

Figure 18, The stall (the top figure is experiencing normal lift, the bottom one has stalled)

The next image illustrates an actual aerofoil (in a wind-tunnel with smoke stream-lines) in stall – note the separated flow forming vortexes in the separated area.



(Creative Commons image "1915ca abger fluegel" author unknown (out of copyright image from 1915), license: CC BY-SA 3.0)

Figure 19, An actual wing in stall

It was mentioned earlier, that a laminar boundary-layer results in less drag. However, a turbulent boundary-layer keeps the airflow attached to the aerofoil for longer - and therefore delays the stall (which is obviously usually desirable). So these two issues sometimes conflict with each other in aerofoil design.

Another way of looking at stall is that, at high angles of attack, the pressure on the upper surface of the aerofoil cannot recover to the ambient pressure by the trailing edge - and this *adverse pressure gradient* causes the flow to separate.

It can be shown that an aircraft has a stall-speed, below which it cannot travel without stalling and this is given by:

$$v_{stall} = \sqrt{\frac{2\left({}^{W}/_{A}\right)}{\rho C_{L(max)}}}$$

Where W/A is the wing loading and $C_{L(max)}$ is the maximum value of lift coefficient (just before the stall).

TASK 8

A light aircraft has a mass of 300 kg and a wing area of 16.6 m^2. If it uses a wing with the characteristics shown in figure 17, what is its stall-speed in kph?

Can you reason where this equation comes from (starting from the lift equation).

TOPIC 7 – THE AEROFOIL IN 3D

So far we have just considered the performance of the aerofoil as a cross-section (in two-dimensions). However, real aerofoils are of course three dimensional. This adds some complications to the calculations as the equations given so far assume that the aerofoil is infinitely long.

In reality the flow is disrupted at the attachment point of the aerofoil to a structure (for example to the fuselage in an aircraft or the hub in a wind-turbine). To take the aircraft example further, the flow along the fuselage mixes and interferes with that across the wing resulting in interference drag (which we mentioned earlier).

At the other end of the aerofoil another effect dominates. We already know that the upper surface of the wing has a lower pressure than the lower surface. This causes the air from the bottom to "spill" around the end (tip) of the wing as it tries to move from high to low pressure as shown.

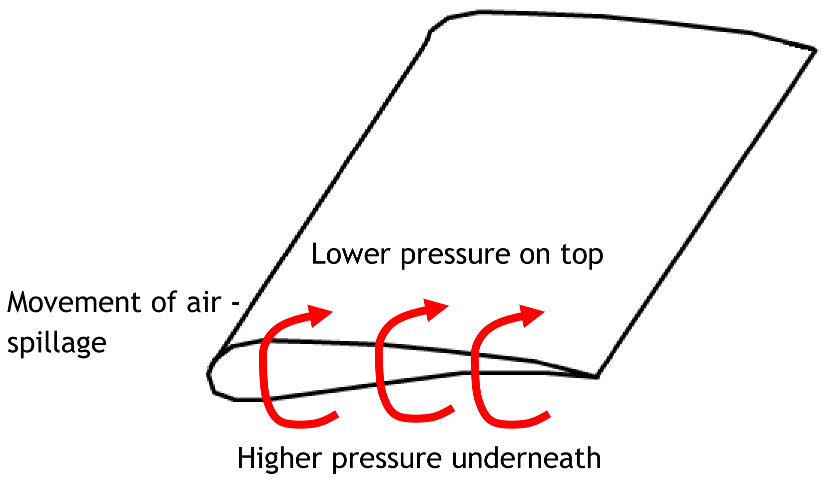


Figure 20, Diagram of spillage from high to low pressure over the aerofoil tip

As the wing moves forward though the air this circulation causes a vortex to form behind the airfoil as shown in the diagrams on the next page. The vortex is an energy loss and therefore causes drag. This drag is the induced drag we mentioned before. The vortexes generated can be so powerful that, in planes, they can cause severe turbulence to following aircraft.

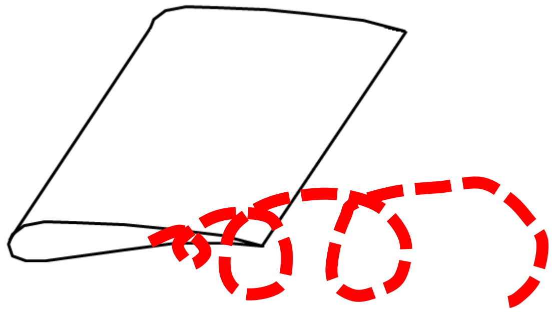


Figure 21, Wingtip vortex

(Image "lift induced vortices" by NASA, license: NASA terms and conditions)

Figure 22, Wingtip vortex made visible by plane flying through cloud

The winglets often fitted at the end of the wings in modern aircraft are designed to reduce the effect of these vortices as shown overleaf.

(Image “Learjet 28-29” by NASA, license: NASA terms and conditions)

Figure 23, A Learjet aircraft with winglets

Aircraft wings also need other aerodynamic control structures like flaps, ailerons and airbrakes. These must also be shaped to distribute the lift loads in a structurally sound way along the length of the wing - and also to ensure that when they stall, this happens first at the root of the wing and away from the control surfaces. All this complicates the three dimensional design.

TOPIC 8 – THEORETICAL METHODS OF LIFT CALCULATION - CIRCULATION, VORTICITY AND LIFT

So far we have covered simple calculations on aerofoils – but what about the detailed design of an aerofoil, how is this done?

The earliest aerofoils were developed purely through experimental trial and error. Since the advent of modern computing in the 1970s and 80s, the design is done with the aid of Computational Fluid Dynamic (CFD) models - which we’ll discuss in the next section.

In-between these two approaches, theoretical techniques based on flow circulation were used – and we'll discuss these in this section. These methods are still used by aerodynamicists for initial and rough calculations, often before the design is submitted for rigorous CFD evaluation. Unfortunately however, the procedures are mathematical and tedious and we'll only cover the basics here – a detailed treatment is providence of specialist aerospace and masters' degrees.

Circulation and lift.

Consider the flow of fluid around an aerofoil. As we've already discussed, the flow over the top surface is faster than that along the bottom. Because of this, the fluid can be thought of as circulating in some way around the aerofoil as shown.

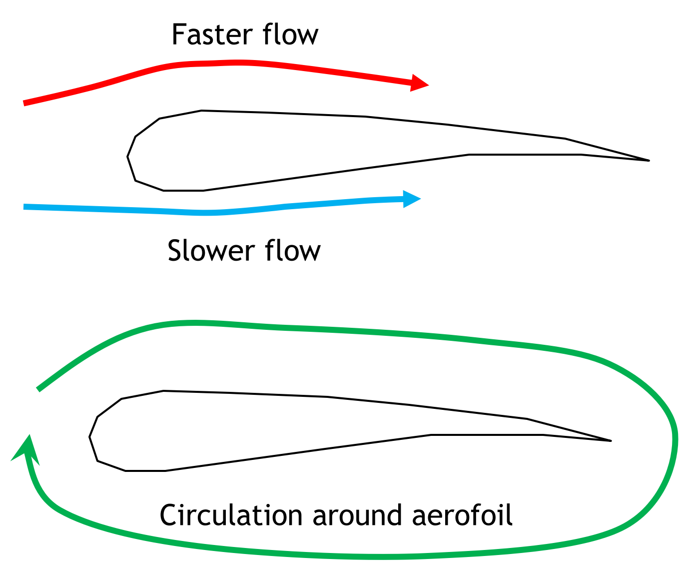


Figure 24, The idea behind circulation

In fact you can show that the actual flow over the aerofoil is the sum of this circulation element and the plain-flow approaching the wing as shown overleaf.

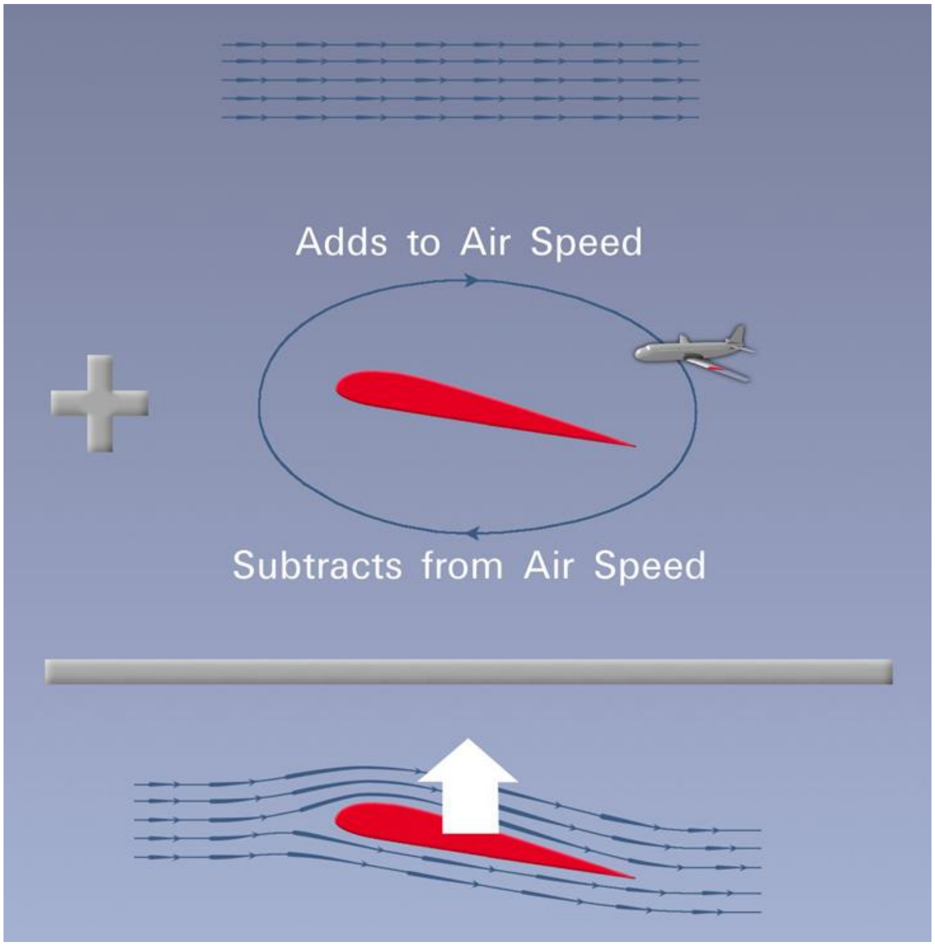


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Figure 25, Circulation adds to plain flow to generate the flow around the aerofoil

Because angular momentum is always conserved, when an aerofoil starts moving and develops its circulation, it leaves a stationary vortex behind it in the fluid called the "*starting vortex*" (technically this is called "shedding the starting vortex") – this vortex is equal and opposite to the circulation around the aerofoil (and similarly, when it stops, it sheds another called the stopping vortex).

Now here's the important point: In the early 20th century, the German scientist Martin Kutta and the Russian Nikolai Joukowski showed that lift was related to circulation by the following equation:

$$L' = \rho V_\infty \Gamma$$

This governing equation relates lift per unit (wing) span L' (N/m) to the fluid density ρ (kg/m^3), free-stream velocity magnitude V_∞ (m/s) and circulation Γ (m^2/s).

This is the basis of all the methods which relate circulation to lift and is called the ***Kutta-Joukowski Theorem***.

TASK 9

What is the circulation around the aerofoil described in task 5 (you can assume the wing-span is 10m)?

The problem is now just one of calculating the circulation Γ around the aerofoil (and this is the bit which unfortunately proves not to be quite so simple).

Circulation is defined mathematically as the line-integral of the velocity $\boldsymbol{V}$ (vector) around a closed curve in the flow – like that illustrated in green in figure 24 ($d\boldsymbol{l}$ is a small length segment of the curve):

$$\Gamma = \oint_C \boldsymbol{V}.d\boldsymbol{l}$$

It is also related to the vorticity of the flow as it is the sum (integral) of all the vorticity (ω) in an area A:

$$\Gamma = \iint_A \omega.d\boldsymbol{A}$$

Vorticity itself is just a measure of the amount of angular velocity (or spin if you like) of the flow. It has a numerical value of twice the total angular velocity of the flow, but from a mathematical point of view is the Curl of the fluid velocity:

$$\boldsymbol{\omega} = \nabla \times \boldsymbol{V} = Curl\, \boldsymbol{V}$$

For flow over a flat plate, it turns out to be the difference between the velocities over the upper and lower surface. Vorticity can be measured experimentally using a vorticity meter.

This is just a body which can be placed in the fluid with attached vanes which allow it to turn in the presence of vorticity – an arrow on the body indicates angle of rotation.

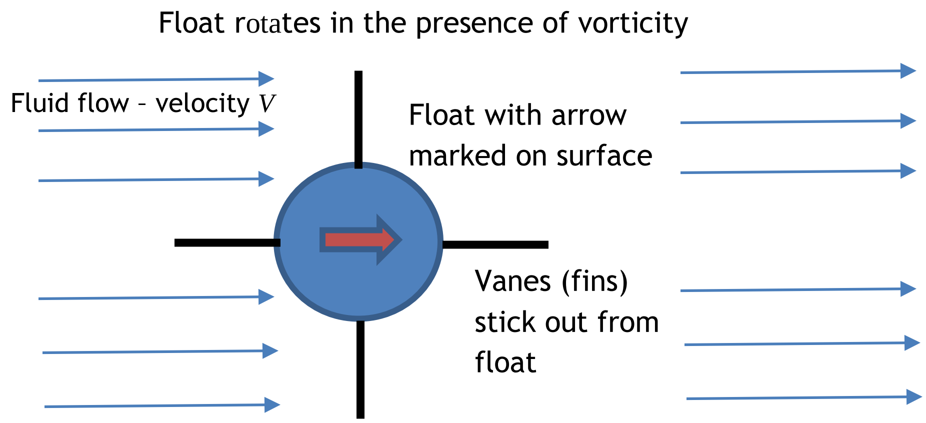


Figure 26, A floating rotating-vane voracity meter – looking down from above

There are two main methods which calculate aerofoil parameters, using these ideas. These are: *Thin Aerofoil Theory* and *Lifting Line Theory*. Let's look at these one at a time.

1. Thin aerofoil theory.

Thin aerofoil theory is perhaps the simplest of the methods. It can calculate lift and centre of pressure position, but unfortunately not drag or stall angle.

The method works by assuming that the wing is thin (the thickness is small in relation to its chord length – hence the name). In fact, the whole wing is represented by its camber-line only. This in turn can be described by an equation in the x coordinate $Z(x)$ which we've already discussed (see task 2). The camber-line can then be thought-of as being representative of a stream-line in the fluid. We can imagine vortices around this camber-line as shown overleaf - the sum (integral) of which is the circulation (camber-line in red, vortices in blue).

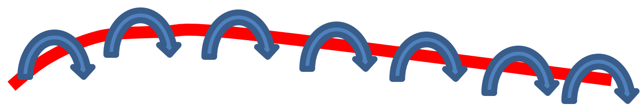


Figure 27, Vortices about the camber-line

The strength of the vortices and therefore the circulation can be calculated using Fourier analysis. I have put a basic description of this in appendix A – your tutor will indicate to you whether this is examinable or not.

Some important results from thin aerofoil theory:

By working through the thin aerofoil theory some important and general results emerge. These include:

a) Symmetrical aerofoils:
   The circulation of a symmetrical aerofoil is given by:

$$\Gamma = 2\pi\alpha V_\infty \frac{c}{2}$$

   And the centre of pressure is located at the ¼ chord point (quarter of the way along from the leading edge)

b) Asymmetrical aerofoils:
   The lift coefficient for an asymmetrical aerofoil is given by:

$$C_L = 2\pi(\alpha - \alpha_{L=0})$$

   Where $\alpha_{L=0}$ is the attack angle for zero lift. This can be found from:

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^\pi \frac{dZ(x)}{dx}(\cos\theta - 1)d\theta$$

Another way of stating the same result is:

$$C_L = C_{L(\alpha=0)} + 2\pi\alpha$$

Note that, in all these formulae, α is in radians, the symbols have the same meanings as in the previous pages and V_∞ is the free-stream velocity – in our case the same as the relative wind.

TASK 10

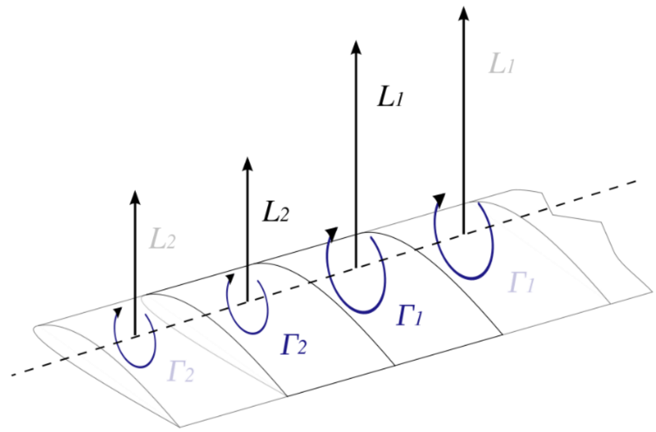
a) Calculate the lift of a symmetrical aerofoil of span 10m, with a 1m chord at 200 kph, sea level with an angle of attack of 15^0. The aerofoil has a square plan shape.

b) Calculate the lift on a cambered aerofoil of the same size and angle, if the lift coefficient at zero angle of attack is 0.5.

c) In both cases, what is the slope of the lift graph?

2. Lifting line theory

This method is also known as *Prandtl lifting-line theory* or *Prandtl-Lanchester lifting line theory*. It can model a finite (3D) wing by representing it as a line of vortices along the wing which decrease in strength (because at the wing-tip, the vortex is shed) as shown overleaf. From this the circulation and hence the lift can be calculated.



(Creative Commons image "lifting-line theory illustration" by Oliver Claynen, license: CC0)

Figure 28, The basis of Lifting Line Theory

Panel Codes

Another method you may also hear of are *Panel Codes* (also called *Aerodynamic Potential-Flow Codes*) – these are more complex computational models, with some similarities to CFD, which calculate parameters by replacing the aerofoil with sources and sinks of fluid. They have some advantages over other computer based models (they are computationally efficient), but are being generally replaced by standard CFD methods.

TOPIC 9 – CFD AND AEROFOILS

The total force on an aerofoil is the sum of all the forces on its surface produced by the pressure distribution on that surface ($P = F/A$). Because CFD can calculate the pressure at all points on the aerofoil, its relatively easy to use it to calculate the total force and therefore both lift and drag. This can be done manually by splitting up the surface into small grid squares of known area and adding the force contribution from each - or more commonly through a lift calculation algorithm built into the CFD solver. CFD has largely replaced the earlier methods of calculation. Shown in the diagram overleaf is the pressure distribution on the surface of a 2D aerofoil generated by a CFD solver – notice the low pressure at the top of the aerofoil and the high beneath – as predicted.

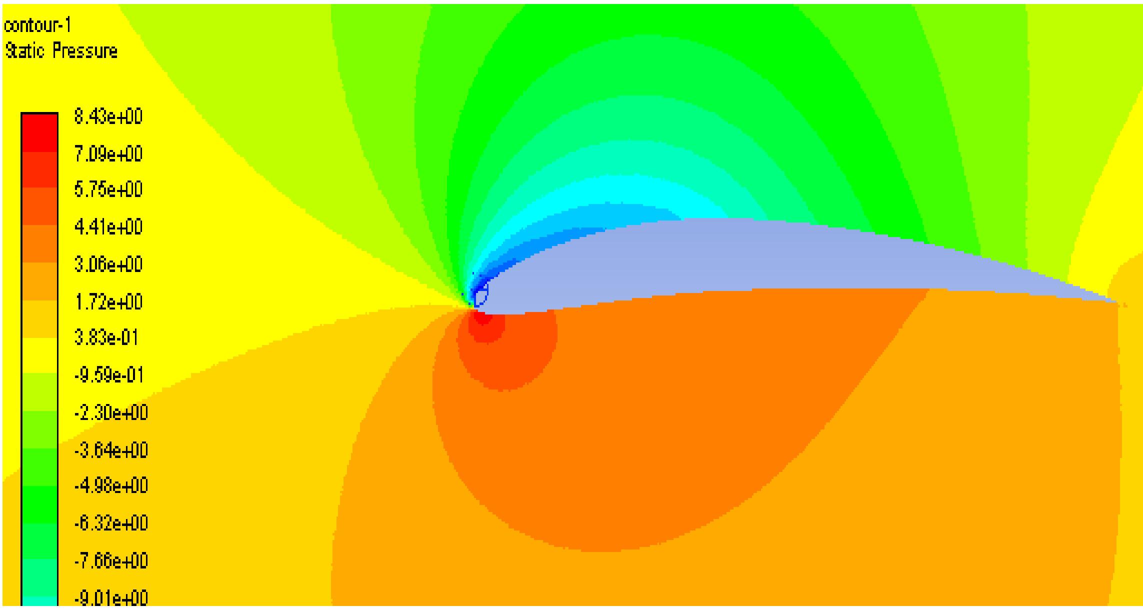


Figure 29, The static pressure distribution around an aerofoil section

TOPIC 10 – SUPERSONIC AEROFOILS

Supersonic aerofoils are based on completely different principles to subsonic types. The idea here is produce high and low pressure regions on the wing using the properties of shockwaves. At supersonic speeds even a flat plate will result in lift as shown (check the pressures in the diagram below with your notes on shockwaves). The plate is shown in black, shockwaves in red, expansion fans in blue and flow in green.

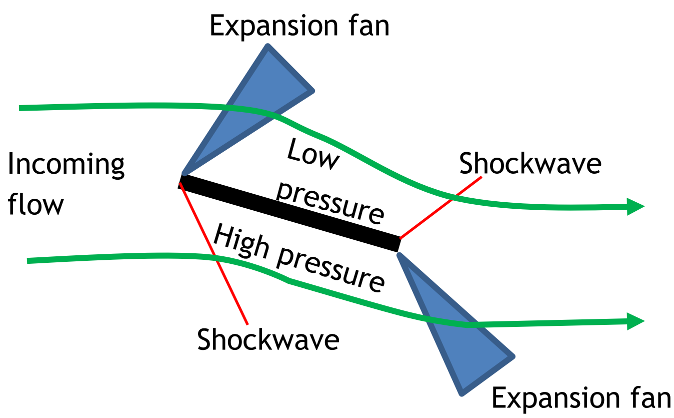


Figure 30, A flat-plate supersonic aerofoil.

Thin-lens and double-wedge shapes are popular supersonic aerofoils.

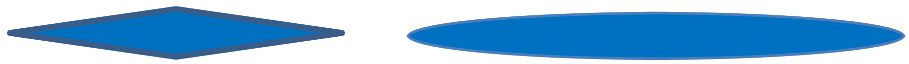


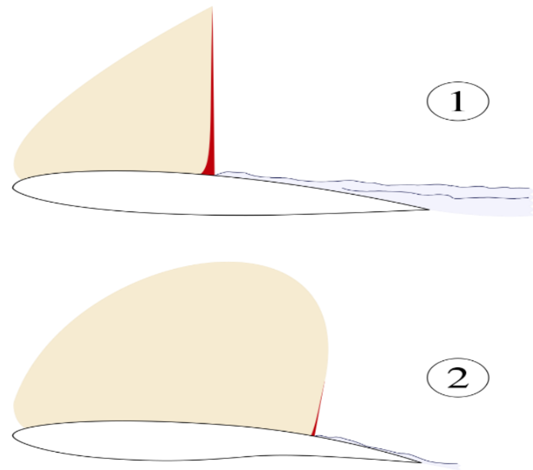
Figure 31, Double-wedge (left) and thin-lens (right) aerofoils.

A major problem with supersonic flight is that such wing designs are pretty hopeless at low speeds, so performance much below the speed of sound is terrible and take-off and landing in such aircraft is problematic as their approach speed to be high and their handling is poor.

Supersonic aerofoils also exhibit *wave-drag* – drag due to the presence of shockwaves (which being irreversible (viscous) phenomena have high losses associated with the increase in entropy they induce in the flow).

TOPIC 11 – TRANSONIC AEROFOILS

Modern passenger aircraft travel at close to the speed of sound. Because air speeds-up over the top of the wing, this typically means that part of the flow is supersonic and part is subsonic. Such flow is called *transonic* – and such mixed conditions make the calculation of flow parameters particularly difficult. However, despite these difficulties, such conditions make for particularly efficient flight and so transonic aerofoil design is a very important aspect of modern aerodynamics. The most common type of aerofoil developed for this region is call the *super-critical aerofoil* and its characteristics are shown overleaf. Such aerofoils are designed to maximize efficiency by delaying and controlling the on-set of shockwaves and hence wave-drag.

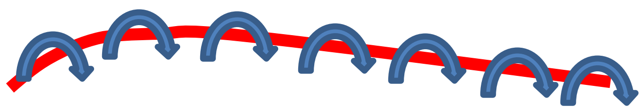


Shockwaves shown in red – note the much smaller wake on the supercritical

Figure 32, Normal (1) and supercritical (2) aerofoils

APPENDIX A – THIN AEROFOIL THEORY

The whole wing is represented by its camber-line only. This in turn can be described by an equation in the x coordinate which we'll call $Z(x)$ or just Z for short. We can imagine vortices around this camber-line the sum (integral) of which is the circulation (camber-line in red, vortices in blue):



It turns out (after much playing with the equations), that it's possible to find the circulation using Fourier transforms of the camber-line equation. In fact:

$$\Gamma = cV_\infty\left(\pi A_0 + \frac{\pi}{2}A_1\right)$$

Where c is the chord length (m) and V_∞ is the free-stream velocity. A_0 and A_1 are the Fourier coefficients of the derivative of the camber-line with respect to distance (x), as shown below. It makes the calculation easier if C is expressed in angular measure (θ) using the transformation shown below ($Z(x)$ is normally differentiated first and then transformed into θ for the integration):

$$A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dZ}{dx}d\theta$$

Where a is the angle of attack

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{dZ}{dx}cos\ n\theta d\theta$$

Where:

$$x = \frac{c}{2}(1 - cos\theta)$$

The pitching coefficient around the ¼ chord point is given by:

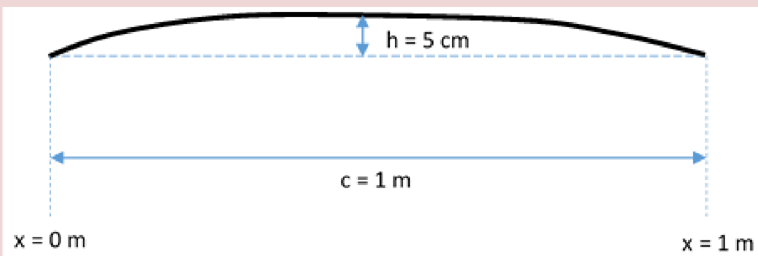
$$C_{m(c/4)} = \frac{\pi}{4}(A_2 - A_1)$$

TASK 11

a) *Show from the equations on the previous page that for a symmetrical aerofoil*

$$C_L = 2\pi\alpha$$

b) *Consider an aerofoil, which is represented by a parabolic camber-line as shown:*



This camber-line has the general formula:

$$Z(x) = -4hx^2 + 4hx$$

i) *Work out an expression for the circulation and*

ii) *The pitching moment for this aerofoil*

Hints: Calculate the derivative before changing variable for the integrations. Some useful formula for use in integrations are shown below (you may have to look for some others):

$$cos^2\theta = \frac{1 + cos2\theta}{2}$$

$$\int cos^3\theta d\theta = sin\theta - \frac{sin^3\theta}{3} + C$$

SUMMARY

- Lift can be viewed as resulting from the distribution of pressure across the aerofoil surface or from the change in momentum imparted to the fluid flow by the aerofoil.
- Aerofoils are defined mainly by their chord, camber-line and thickness – these two latter parameters can be written as mathematical functions of the position along the aerofoil.
- The total force due to fluid on the aerofoil can be split into drag and lift - which are two forces in orthogonal directions.
- Drag has several components: Form, Skin, Induced, Interference and (in supersonic aerofoils) Wave
- A turning moment is induced on an aerofoil if the centre of pressure and centre of gravity are not co-incident.
- Coefficients can be defined to describe Lift, Drag and Moment on an Aerofoil.
- Lift and drag are often plotted on graphs.
- Stall occurs when the fluid separates from the aerofoil.
- Induced drag is due to lift and is caused by wing-tip vortices.
- Three methods of calculating airfoil parameters theoretically are: Thin-aerofoil theory, Lifting-line theory and Panel codes.
- Thin-airfoil theory and lifting-line theory are based on circulation around the aerofoil and the Kutta-Joukowski Theorem.
- Useful general results about symmetric and cambered aerofoils can be derived from these theoretical methods.
- Modern aerofoil simulation and design makes extensive use of CFD methods
- Supersonic aerofoils use the properties of shockwaves to generate pressure differences and hence lift.
- Supersonic aerofoils often have poor low speed performance.
- Transonic aerofoils need to be designed to comply with the demands of both supersonic and subsonic flow.
- A common transonic aerofoil is the super-critical aerofoil.

REFERENCES AND BIBLIOGRAPHY

You may wish particularly to read the appropriate sections in the “Fundamentals of Aerodynamics” by Anderson (and Anderson’s other aeronautical books) and “Aerodynamics for Engineering Students” by Houghton and Carpenter. There are also many YouTube videos and Wikipedia entries illustrating many of the important concepts.

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