FLUIDS 1

PRESSURE

This section discusses pressure in engineering system – what it is, how to calculate it and how measure it.

FLUIDS 1

PRESSURE

OVERVIEW

In this section we will cover:

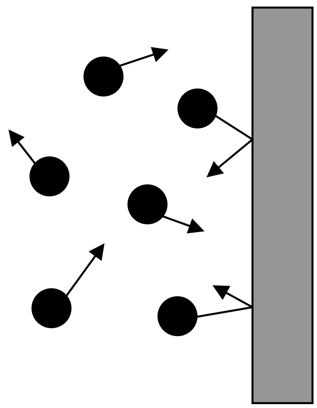
- The nature of pressure
- The difference between static, dynamic and total pressure
- Absolute and gauge pressures
- Hydrostatic pressures in fluids
- The hydrostatic paradox
- Pressures on submerged objects
- Pressure measurement

OBJECTIVE

The concept of pressure in absolutely central to engineering systems – both in fluid mechanics and in thermodynamics. This section explores it detail.

TOPIC 1 – THE NATURE OF PRESSURE

Pressure is caused by the molecules of a fluid (which are in constant motion, due to their temperature) bouncing off the wall of their container or another object.



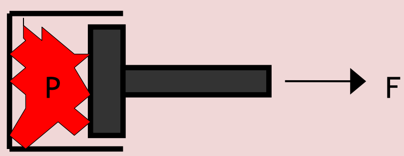
Pressure is the force exerted on the wall per until area.

Pressure: $$p = \frac{F}{A}$$

If force is measured in Newtons (N) and area in square meters (m^2), then pressure is measured in *Pascals*. 1 Pascal (Pa) = $1Nm^{-2}$. Another, sometimes more convenient, measure of pressure is to say that the air-pressure at sea level is 1 *bar*. 1 bar is 101×10^3 Pa (this is also sometimes known as 1 atmosphere or 1 atm).

TASK 1

A mixture of air and petrol explodes in an engine cylinder as show below:



The peak pressure generated is 20 bar and the piston diameter is 10cm, calculate the maximum force produced on the piston connecting rod.

TOPIC 2 – STATIC AND DYNAMIC PRESSURE

The pressure measured in still air (or fluid) is called *Static Pressure*. If an object is moving, the pressure on it is greater because, apart from the speed of molecules, there's also the extra speed of the object to consider (think of sticking your hand out a car window while travelling fast). The same argument also applies to a stationary object in a travelling fluid stream (like a tree in the wind).

This extra pressure, caused by movement, is called *Dynamic Pressure*. The Dynamic Pressure is given by:

Dynamic Pressure: $$p_D = \frac{1}{2}\rho v^2$$

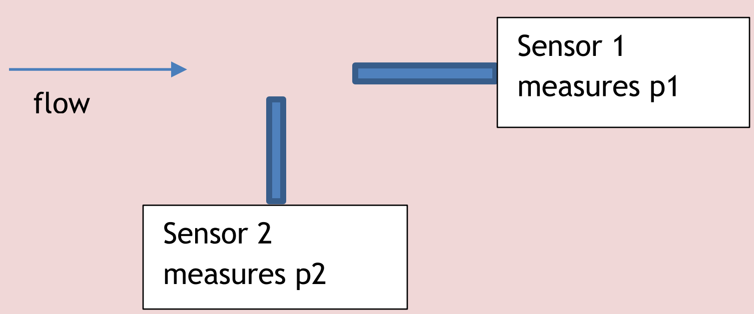
Where ρ is the density of fluid and v is its velocity. Notice the similarity between this formula and that for Kinetic Energy ($0.5mv^2$) – this is because pressure is energy per unit volume (per m^3) – we can therefore consider static pressure as a measure of potential energy and dynamic pressure as a measure of the kinetic energy in the flow.

The *Total Pressure* on an object is the sum of static and dynamic pressure (plus the hydrostatic pressure, if this is differentiated from the static pressure - see section 4):

$$p_{total} = p_{static} + p_{dynamic}$$

TASK 2

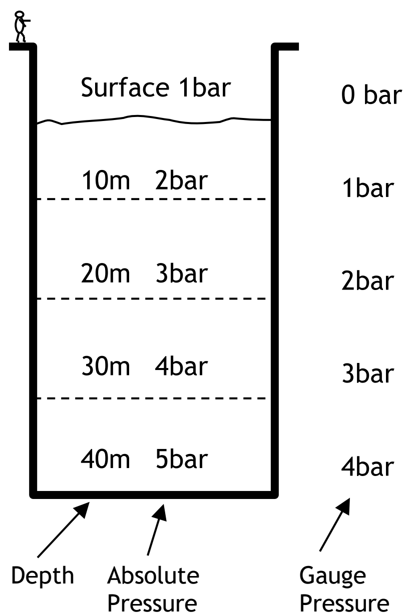
1. *A person sticks their hand out of a car window at 100km/h. Assume that the hand measures 5cm x 10cm.*
    a. *What is the static, dynamic and total pressure on their hand?*
    b. *What is the force on their hand due to dynamic pressure?*
    c. *Why is no force felt due to the static pressure?*
2. *Two sensors measure pressure as shown below: p1 is the pressure measured by a sensor facing into the flow. p2 is the pressure measured by a sensor at right angles to the flow. Write down expressions for the static, dynamic and total pressures.*



You may also see the term “stagnation pressure” used in some books. This is essentially the same as the total pressure defined above (it’s defined as the (static) pressure measured when a moving flow has been brought to a complete to a complete halt).

TOPIC 3 – GAUGE AND ABSOLUTE PRESSURES

Pressure is measured using a pressure gauge or sensor (of which there are several different types). To illustrate how pressure varies in water and how it is measured, consider a deep swimming pool as shown below.



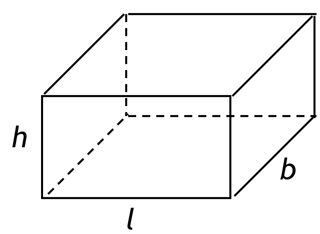
The pressure of air at the top of the pool is 1 bar (atmospheric pressure). Each time you go down 10m, the pressure increases by a further 1 bar (we’ll see why in the next section). So at 10m the pressure is 1 bar (from the air) + 1 bar (from the 10m of water) = 2 bar. This is called the *absolute pressure*.

However, if we had a pressure gauge, we’d probably zero it at the top (so atmospheric pressure, which is after all the ambient pressure which surrounds us all the time, reads as zero). Then when it gets down to 10m it’ll read 1 bar, due to the 10m of water above us. This is called *gauge pressure* for obvious reasons.

TOPIC 4 – HYDROSTATIC PRESSURE - THE RELATIONSHIP BETWEEN PRESSURE AND DEPTH IN A FLUID

In the previous section we saw that static pressure increased as we descended in a swimming pool – and you are probably aware that pressure increases in the ocean with depth or decreases with height in an aeroplane. Loosely speaking this is due to the weight of the fluid above you in each case. We can write a simple formula to express the relationship between pressure and depth in a fluid. Given below is a straight-foreword derivation of this; however the derivation isn't really very rigorous – for a better one, see Fluid Mechanics by Douglas et al, 4th ed, chapter 2.

Consider a box of length l, height h and breadth b.



Now remember that *Pressure* = $\frac{Force}{Area}$

If the force is acting on the bottom of the box, then the area is lb. The force due to the weight of fluid in the box $mg = \rho g V$ (where V is the volume). However, the volume of the box is lbh and so.

$$Pressure = \frac{F}{A} = \frac{\rho \times g \times lbh}{lb} = \rho g h$$

Hydrostatic pressure: $p = \rho g h$

So, the pressure in a fluid is the fluid density (ρ), times the acceleration due to gravity (g), times the depth of the fluid (h). Again, notice the similarity between this formula and that of gravitational potential energy (mgh) – this is the potential energy due to pressure measured per m^3.

This type of static pressure is called *Hydrostatic Pressure*. The reason for differentiating this from "normal" static pressure is that static pressure (pressure in a still fluid) might be a result of other factors (like a pressurized gas cylinder for example). On the other hand, hydrostatic pressure results from gravity acting to give a fluid weight.

Notice, from the formula, that because g and ρ are constant, the pressure in water varies directly with depth. This means that you could use the depth of water as another way of specifying pressure. For example 1 bar (101 x 10^3 Pa) is equal to 10m of water and 2 bar is 20m. This idea of using depth of water as a way of specifying pressure is often used by engineers and is called *head*. We say that a pressure of 1.5 bar is a "Head of 15m" (it's the same pressure as would be exerted by a 15m high water-column).

TASK 3

1. *What is the absolute and gauge pressure in Bar and Pascals in:*
   a. *56m of still water.*
   b. *56m of water of water moving because a current at $2ms^{-1}$. In this case specify the static, dynamic and total pressure. Specify the dynamic pressure in meters of head.*
   c. *In the case of part b. above, what force would be caused by the dynamic pressure on a 10cm diameter disk placed at right angles to the flow.*
2. *What is the hydrostatic pressure at a depth of 3m in mercury? Specify your answer in Pascals, Bar and meters of Head. What would be the pressure in the same situation on the surface of the moon (just in Pa)?*

Sometimes (for example in the case of pressurized system) it is convenient to separate hydrostatic pressure from the static pressure due to other factors – in this case:

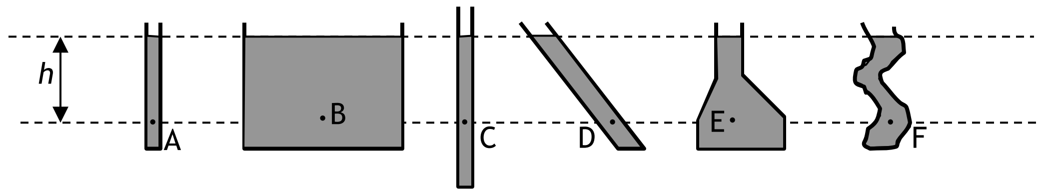
$$p_{total} = p_{static} + p_{hydrostatic} + p_{dynamic}$$

Normal air pressure at sea-level is also the result of hydrostatic pressure – in this case the weight of the atmospheric air above us.

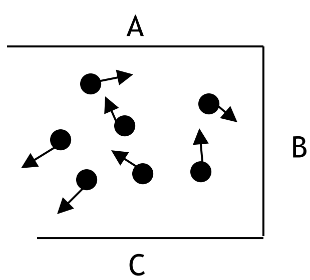
TOPIC 5 – THE HYDROSTATIC PARADOX

There are several interesting things to notice about the hydrostatic equation given in the section above.

Firstly, the size of the container doesn't matter (l and b were cancelled out of the equation). So the hydrostatic pressure at points A, B, C, D, E and F of the diagram below are all the same.

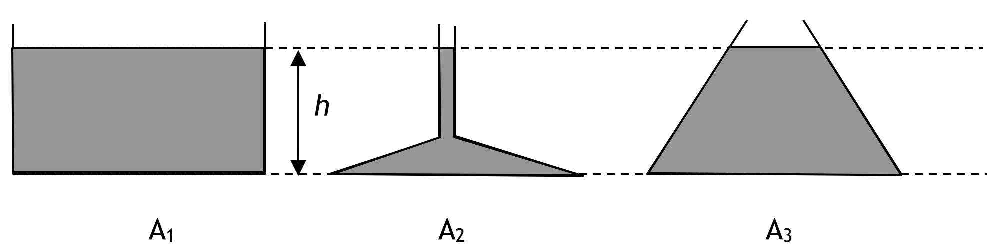


Secondly, the pressure is the same in all directions. This makes sense if you consider that pressure is caused by the many particles "bouncing" of the wall and each other, these particles are moving randomly in every direction.



If walls A, B and C at all at approximately the same depth, then because the particles colliding with them are moving randomly, the pressures would approximately be the same.

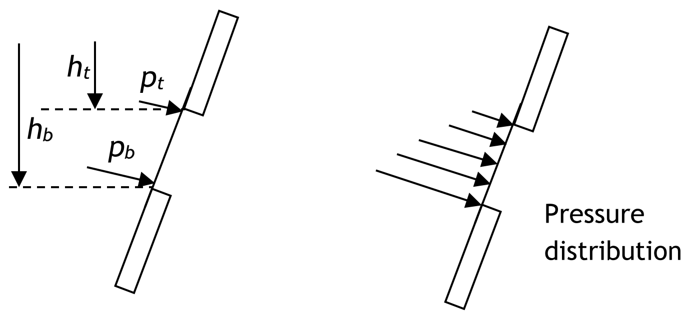
Finally, we come to an interesting point, which is not intuitively obvious and so is sometimes called the *Hydrostatic Paradox*. Consider three containers of liquid as shown below.



If the three areas, A_1, A_2 and A_3 are all the same size then, because the pressure is the same at the bottom of each container (they are all the same height), so is the force! $F=pA$. This doesn't make intuitive sense because the first container contains much more water (and therefore a greater weight of water) than the second or third ones. The reason for this can be seen if one looks at the complete derivation of static pressure – the pressure depends on the column of fluid above the object.

TOPIC 6 – FORCES ON SUBMERGED SURFACES

Consider the underwater window shown below – this has water pressure on one side and a vacuum on the other.



Because the depth of water at the bottom (h_b) is greater than that at the top (h_t), we must be careful to take this into account in any design. To calculate the total force on the window, we have to integrate the force over the whole area (although, to be on the safe side, we could just assume that the force on the bottom was the present over the whole area). For a window like this one, the result turns out to be.

$$F=\rho g h_c A$$

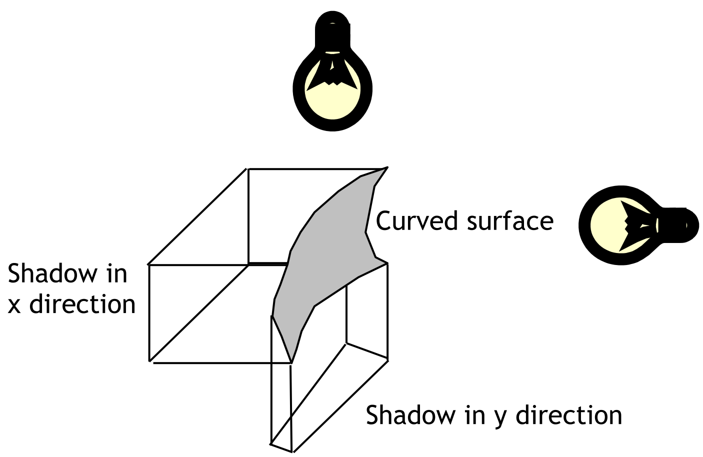
Where h_c is the depth of the window's centroid. To this we must add the extra force on the outside of the window due to the air pressure at the sea-surface (the calculation above is based on the gauge pressure at the window).

There a couple of points to note about this. Firstly, for simple shapes (like rectangles and circles) the centroid is located at the center of the shape – you'll find out later in the course how to calculate it in more complex situations. Secondly, if the other side of the window has another fluid pressing against it, this will exert an opposing force from the other side and the total force will be the difference between them (although if the opposing force is small, it can often be ignored).

TASK 4

An undersea manned habitat has a rectangular transparent window. The top of the window is 9.5m below sea-level and the bottom is 10.5m. The window is 2m in width and is sloping at 45º to the sea-bed (it is sloping upwards, so that the occupants can see the surface). Inside the habitat is air at atmospheric pressure. Calculate the force on the window (do not ignore the internal pressure of the air).

To work out the forces due to pressure on a more complex 3 dimensional curved surface, there is a fairly simple procedure which can be followed. The surface can be projected in the *x* direction onto a flat plane (think of shining a light behind the object and the shadow it would cast). Likewise onto the *y* plane (this time think of the shadow cast be a light above it). These two situations are shown on the diagram below.



Each of these two directions can be treated as a plane (as in the last example), and the force calculated by integrating across it. This gives the force in the *x* direction and the force in the *y* direction. The total force acting on the object is then the resultant.

$$F_{total} = \sqrt{F_x^2 + F_y^2}$$

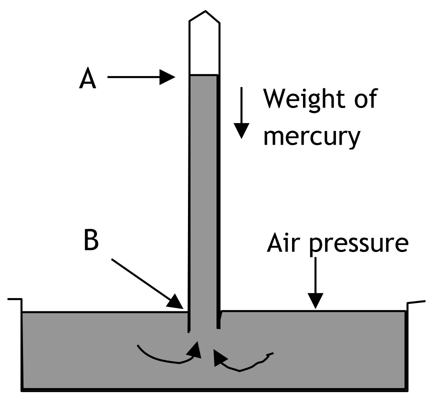
For more detail see Douglas p62 – 71.

TOPIC 7 – PRESSURE MEASUREMENT

Obviously in all the discussions above, being able to measure the pressure accurately is very important. In this section we'll have a look a number of pressure measurement devices and sensors.

i) Manometers

Manometers are primitive devices used for measuring pressures (they are generally not used if accurate readings are required). The *mercury barometer* is a simple type of manometer, which is frequently used to measure atmospheric pressure. This consists of a tube completely filled with mercury. When the tube is turned upside-down and placed in a beaker of mercury, as shown below, there are two forces acting.



The first force is due to the weight of mercury – this drags the mercury down the tube into the beaker. The second force is due to the air pressure on the surface of the mercury in the beaker, which tries to push it back up the tube. Sooner or later the two forces will balance and the mercury will settle at a certain height (marked A on the diagram).

The pressure at point B is the pressure at point A + the pressure due to the weight of the column of mercury. $p_B = p_A + \rho gh$. But since point A is a vacuum.

$$p_B = \rho gh$$

Which is just the air pressure. This equation also applies to all the other manometers discussed below. In inclined tubes, the height h is the vertical distance from the top of the liquid to the bottom, regardless of the angle of the tube.

At sea level the column of mercury turns out to be around 760mm high (or 29.92 inches). So atmospheric pressure is sometimes quoted as 760mm of Hg or 760 *torr* (1 torr = 1mm of Hg). In the old units of *pounds per square inch* this is 14.49. So, to complete our list of units of pressure:

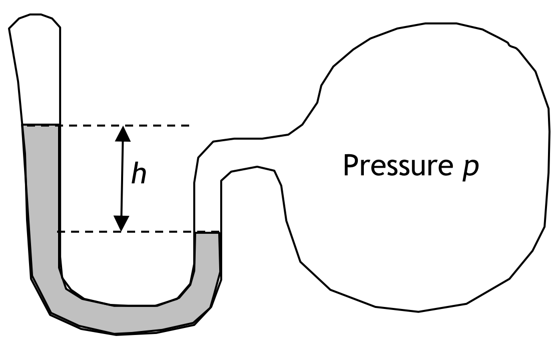
1 barr = 1 atm = 760 torr = 760 mm Hg = 14.69 psi = 29.92 inches Hg = 101325 Pa = 101.325 KPa

TASK 5

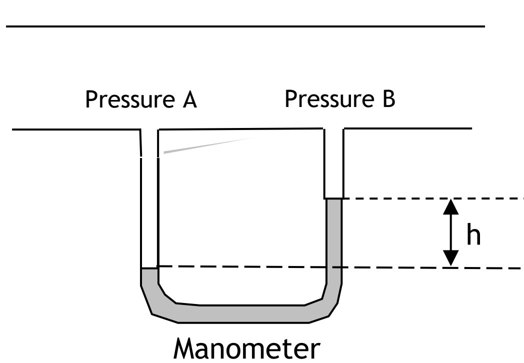
a. Show using the equation that the 760 torr figure is correct.
b. How high would the column be if the fluid were water?

(If you think about part b carefully, you should be able to work out the answer from common sense – but confirm this through calculation).

A manometer can be rigged up to measure any pressure (for example in a pressure-vessel) in mm of Hg, as shown (the tube is often inclined at an angle).



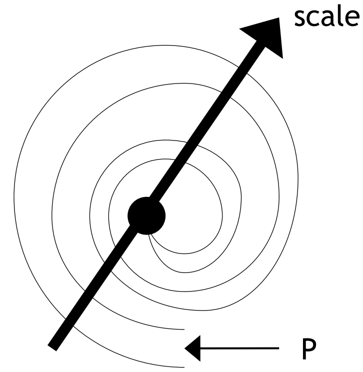
More often in engineering, a manometer is rigged up so read differential pressures (differences in pressure (Δp) between two points), for example in a pipe.



ii) The Bourdon tube or gauge.

This is a simple device which is widely used (for example in home weather-barometers, gas bottle gauges, many “meter style” analogue pressure meters and scuba-bottle gauges). It is simple and reliable but generally not very accurate.

The Bourdon tube is a conical shaped, coiled pipe. The large end of the tube is connected the pressure to be measured and the body is in another fluid (like the ambient air or a vacuum). As the pressure inside increases, the tube uncoils and a pointer at the end rotates to show how much movement there has been. There is usually a gearing mechanism between the tube and the point, not shown in the diagram below:



TASK 6

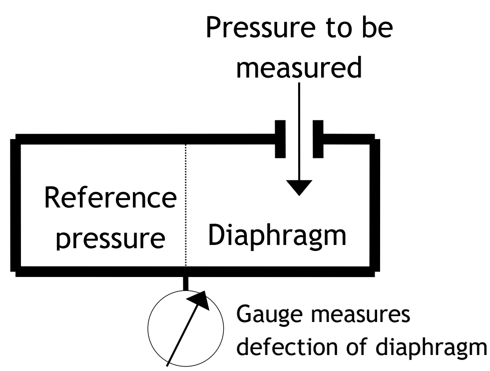
Google “Bourdon tube pressure gauge” and look at some of the images of how the mechanics of the gauge are arranged. Once you have done this, look at the Wikipedia page on “pressure measurement” to see some of the other types of gauge available.

It is also possible to put a movement sensor on the Bourdon gauge, so that pressure can be read off electrically.

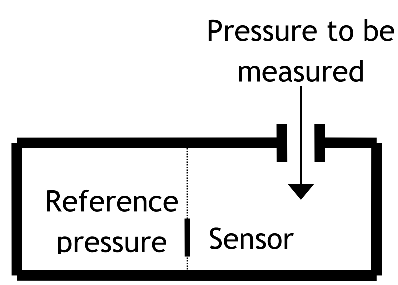
iii) Diaphragm pressure gauges.

Diaphragm (or similar) pressure gauges are quite common – they have similar advantages to the Bourdon gauge but are usually more accurate. They are also the most common gauges for electronic implementation.

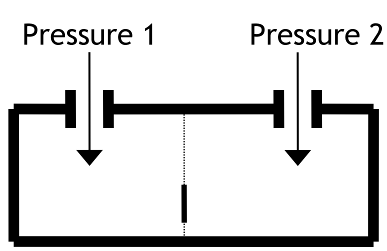
They consist of two chambers – one of which holds fluid at a known pressure (the reference chamber), the other is connected up to the pressure to be measured. A thin diaphragm separates the chambers and this bends and deflects according to how large the pressure difference between the two chambers is – giving a measure of the pressure difference:



Instead of an analogue meter, the deflection of the diaphragm can be measured with an electronic sensor. This can be a simple strain-gauge, a piezoelectric deflection sensor or a capacitive sensor:



Finally this gauge is also easy to modify to measure differential pressure:



SUMMARY

- Pressure is caused by the molecules of a fluid (which are in thermal motion) hitting the sides of the container.
- The molecules impart a force to the container wall when they hit them by conservation of momentum.
- This force, per unit area is pressure. The SI unit of pressure is the Pascal (Pa).
- One Pa is $1N/m^2$
- Atmospheric pressure is 1×10^5 Pa (101 kPa) and is sometimes called one Bar or one atm.
- Extra pressure caused by movement is called dynamic pressure.
- Pressure can also be thought of in terms of energy – Dynamic pressure in terms of Kinetic Energy and Static pressure in terms of Potential energy – Pressure is energy per unit volume.
- Absolute pressure is the true pressure measured on a gauge which reads zero in a vacuum.
- Gauge pressure is the pressure measured on a gauge which reads zero in air at sea-level.
- Hydrostatic pressure is static pressure caused by immersion in fluid given weight by a gravitation field.
- Going down each 10m in water increases hydrostatic pressure by 1 Bar.
- Because of this pressure can be written as "Head" in meters – 10m of head is equal to 1 Bar of pressure.
- When considering hydrostatic pressure, the shape of vessel holding the fluid is unimportant.
- Pressure differences between the inside and outside of objects (like subsea and aerospace structures) causes forces (and therefore stresses) on these objects.
- Three common pressure measurement systems are manometers, Bourdon tubes and diaphragm meters.

REFERENCES AND BIBLIOGRAPHY

Wikipedia entries for the various gauges mentioned are worth reading.