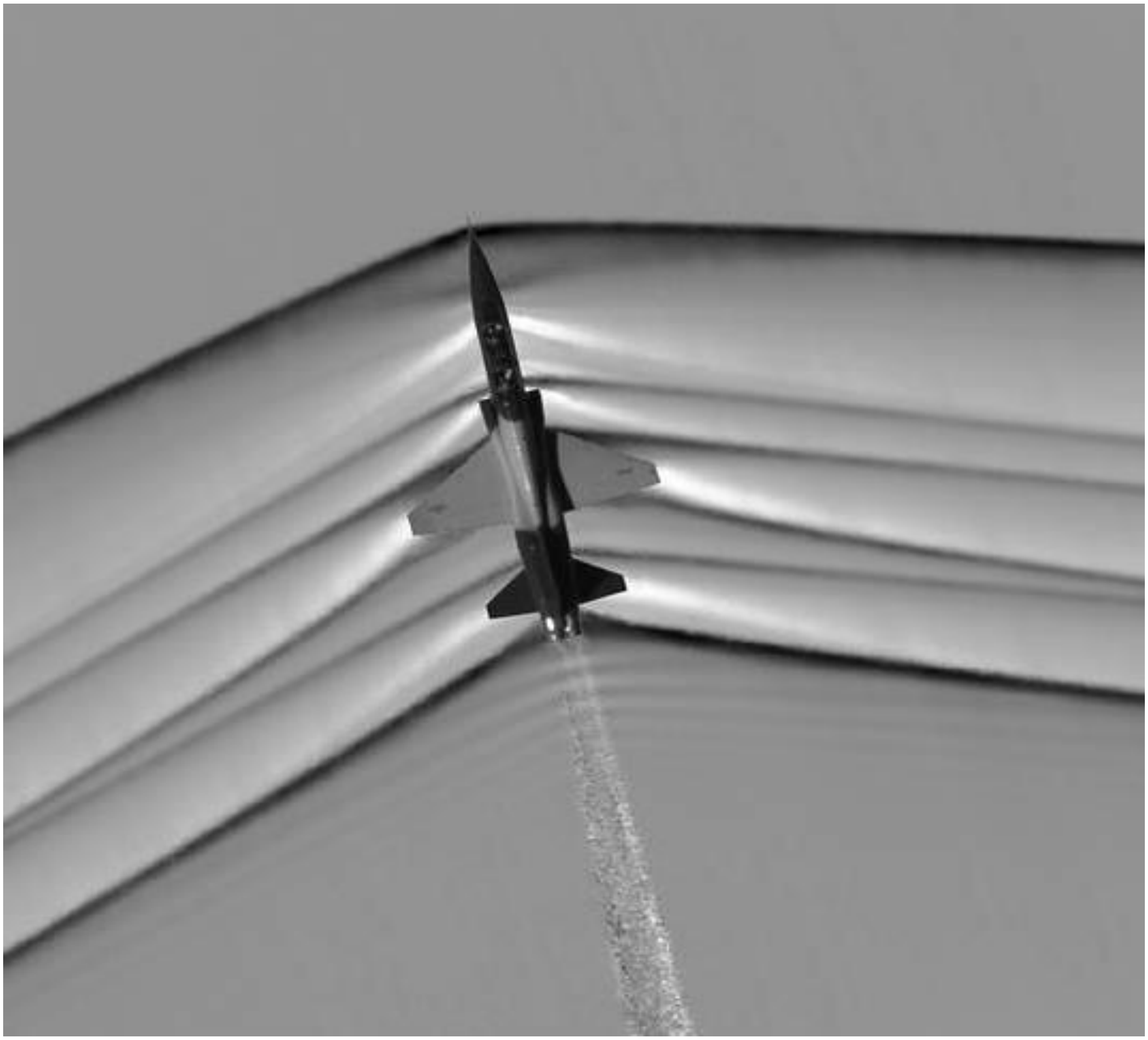


FLUIDS 4

SHOCKWAVES

In this section we will learn about supersonic shockwave systems. The different types of shockwaves and related phenomena, their properties and how to perform calculations involving them.

FLUIDS 4

SHOCKWAVES

OVERVIEW

In this section you will learn about:

- Revision of some shockwave ideas
- The normal shockwave and fluid parameters before and after it
- The oblique shockwave and its effect on fluid parameters
- Use of the θ-β-M graph
- Hypersonic flows and limiting cases
- The Prandtl–Meyer expansion fan

ASSUMED KNOWLEDGE

In this subject it is assumed that you already have knowledge about the following topics:

- *A good knowledge of fluid parameters*
- *Incompressible flow including Bernoulli, Continuity, Reynold's Number and Thrust equations*
- *Isentropic compressible flow including the energy equation and pressure, density and temperature relationships*
- *The basic (qualitative) ideas behind normal and oblique shockwaves*

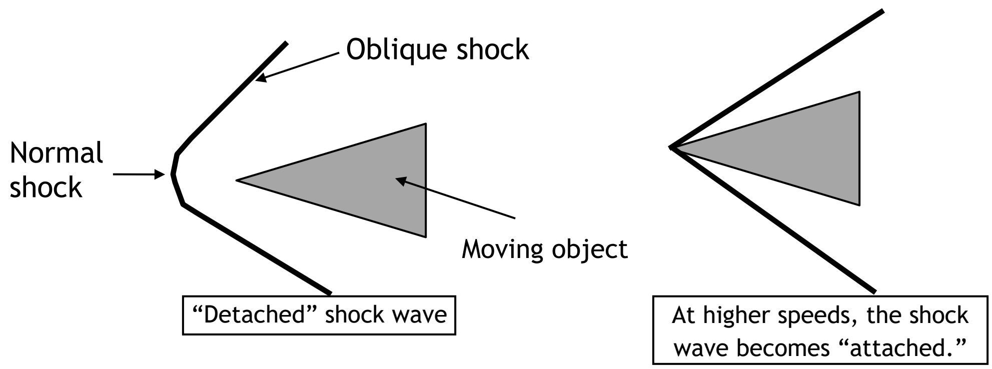
Front cover picture: Shockwaves and expansion fans around a T-38C training supersonic aircraft above the Mojave Desert.

OBJECTIVE

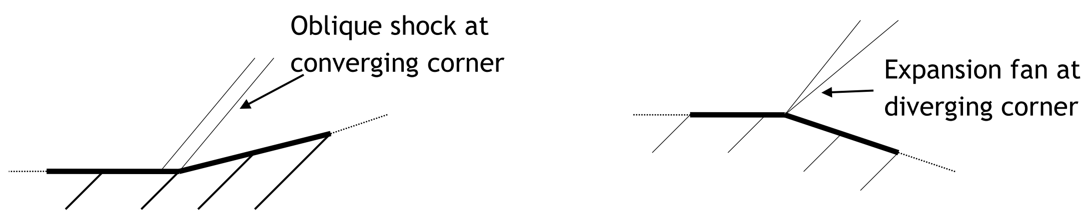
To be able to understand different shockwave and related phenomena and perform calculations regarding them.

TOPIC 1 – SOME REVISION

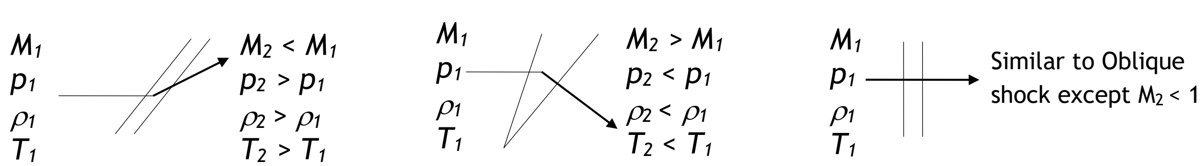
When the speed of a flow (or of an object moving through a flow) exceeds the speed of sound for the fluid, shock waves are formed. These waves tend to come in three forms - “Normal” shocks which are at right angles to the flow, “Oblique” shocks which are at an angle to it. The third type - “Expansion fans” occur in the presence of more complex shapes. The first two types are shown in the examples below:



Expansion fans appear in more complex shapes when the body diverges away from the flow.



Flow across shock-waves is non-isentropic because the parameters change violently and the viscous properties of the fluid cause major thermal effects. The relationship between fluid parameters before and after the shocks are given by standard equations or by tables. The qualitative situation is shown below:



Oblique shock Expansion fan Normal shock

In the following pages, we will explore the different types of shockwaves and expansion fans, one by one.

TOPIC 2 – THE NORMAL SHOCK

In the discussions which follow I am going to assume that you are familiar with the basic physical cause of shockwaves themselves (the bunching on fluid molecules, as an object hits them faster than they can move away, causing a thin (but severe) regional rise in local fluid density). If you are not sure about this, ask your lecturer to explain it before you begin your in-depth study.

Normal Shock Scenarios

Normal shockwaves (NSW) happen when a flow goes from supersonic to subsonic (or vice-versa) and doesn't change its direction. This usually happens in one of two circumstances.

a) In a duct.

If a flow slows down or speeds up in a duct and becomes crosses the Mach 1 barrier, this is accompanied by a NSW. A good example of this is in a rocket engine as shown in figure 1.

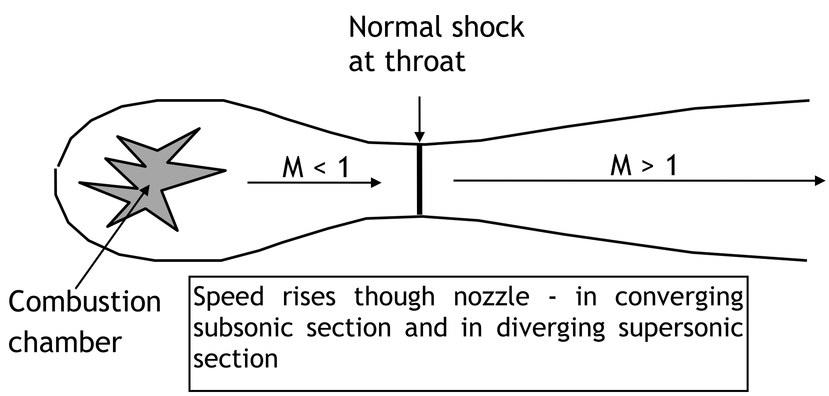


Figure 1, Flow speeding up in a rocket engine (this type of structure is also called a de Laval nozzle).

b) As a detached shockwave.

We've already mentioned this on page 2. A shockwave may become "detached" from a supersonic body because: 1) Its angle of defection is too great for an oblique shock to handle at a particular speed, or 2) The front of the body is blunt and there's nothing to attach to (in this case the shock is sometimes called a "*Bow Shock*"). In these scenarios there's usually a short section of the shock-wave which can be considered as normal and this is attached to an oblique wave as shown in figure 2. In these cases, like all NSWs, there is a "bubble" of subsonic flow behind the area of normal shock.

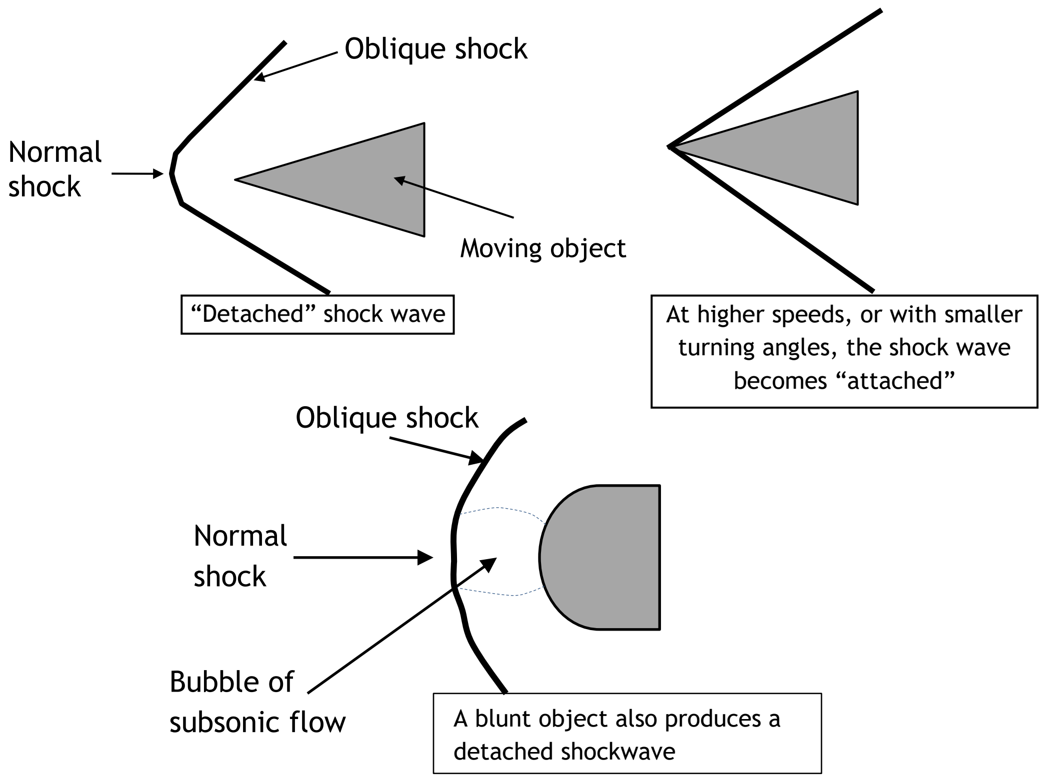


Figure 2, Detached and bow shock-waves.

The normal shock equations

The equations governing the way properties change across a NSW are just derived from standard expressions for the conservation of mass, momentum and energy along with the perfect gas equation. However, the derivations are rather long-winded – they are in all standard aerodynamic textbooks and several explanations are also available YouTube. Because of this I'm just going to quote the results here and then look at how they are used.

Generally we are interested in calculating the parameters downwind of the NSW knowing the upstream parameters. The general situation is shown in figure 3 below:

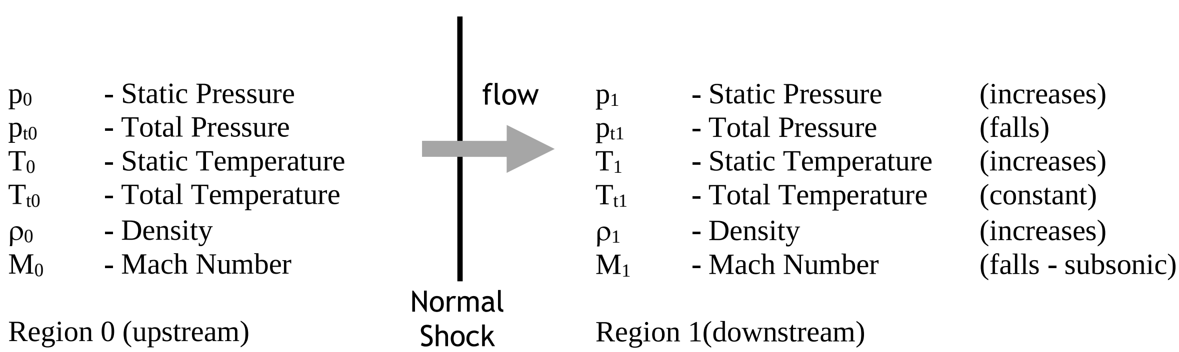


Figure 3, Qualitative flow situation across a normal shockwave.

Here's a reminder of some of the important pressure and temperature parameters shown in figure 3:

- Static pressure - The pressure measured if we were travelling with the flow (or by a manometer at right angles to the flow). Commonly just thought of as "pressure".
- Total Pressure - The static pressure + the dynamic pressure. Pressure felt by object directly in flow or a manometer opening into the flow. Stagnation pressure is the same as total pressure if a ignore the effect of pressure due to height (ρgh) - as is the case in most dynamic flow problems like this. The total pressure would also be the static pressure measured if the fluid were brought to a halt (if the flow were "stagnant").
- Total temperature - Temperature measured at stagnation point (if the fluid were brought to a stop).
- Static Temperature - Measured temperature of flow, moving along with it.

We can now write down the Normal Shock Equations – which give the ratio of parameters before and after the shock:

$$\frac{p_1}{p_0} = \frac{2\gamma M_0^2 - (\gamma - 1)}{\gamma + 1} \qquad \frac{p_{t1}}{p_{t0}} = \left[\frac{(\gamma+1)M_0^2}{(\gamma-1)M_0^2 + 2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{(\gamma+1)}{2\gamma M_0^2 - (\gamma-1)}\right]^{\frac{1}{\gamma-1}}$$

$$\frac{T_1}{T_0} = \frac{(2\gamma M_0^2 - (\gamma-1))((\gamma-1)M_0^2 + 2)}{(\gamma+1)^2 M_0^2} \qquad T_{t1} = T_{t0}$$

$$\frac{\rho_1}{\rho_0} = \frac{(\gamma+1)M_0^2}{(\gamma-1)M_0^2 + 2} \qquad M_1^2 = \frac{(\gamma-1)M_0^2 + 2}{2\gamma M_0^2 - (\gamma-1)}$$

TASK 1

A flow is slowing down through a normal shock and has the following upstream parameters: Static pressure = 850 kPa, Static temperature 80°C, Mach No 1.1, Density 1.1 kg/m^3 (γ = 1.4).

Calculate these flow parameters after the shock.

Normal shock parameters are also tabulated in tables called "normal shock tables" and on-line calculators are also available. You can check your results from task 1 using these.

TOPIC 3 – THE OLIQUE SHOCKWAVE

Unlike a normal shock, the speed after an oblique shock wave (OSW) can be supersonic (or subsonic). OSWs occur when a supersonic airflow encounters a sharp-sloping object which turns the flow into itself and the flow is fast enough so that the shock becomes attached to the object (otherwise the shock stands off the front and is a combination of normal and oblique components). The geometry is shown in figure 4.

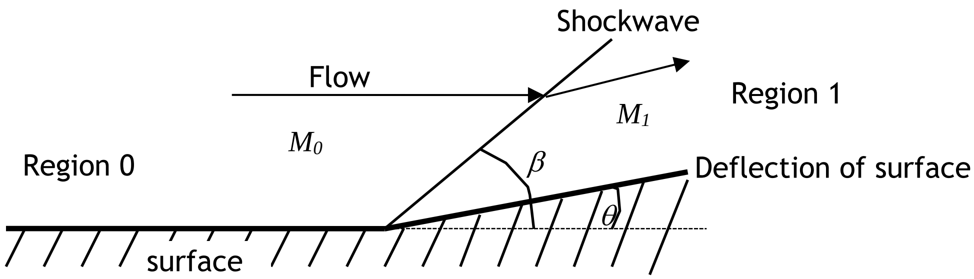


Figure 4, The geometry of an oblique shockwave.

The parameters shown below are designated as for the normal shock.

In general:

- If the flow is external (around a body) M_1 will still be supersonic and the shock is called *weak*.
- If the flow is internal (in a duct) M_1 will be subsonic (due to the downstream back pressure) and the shock is called *strong*.

The relationship between θ-β-M is given by the equation:

$$\tan\theta = 2\cot\beta \frac{M_0^2 \sin^2\beta - 1}{M_0^2(\gamma + \cos 2\beta) + 2}$$

Since this equation does not lend itself to rearrangement it is usually expressed as a graph. This has the form shown in figure 5 overleaf and an accurate version is given in appendix A at the end of the notes. Note that the flow direction is always parallel to the surface.

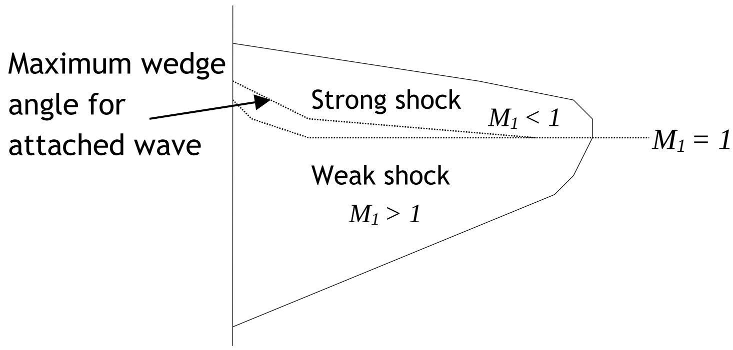


Figure 5, The θ-β-M shockwave graph – see appendix A for useable version

Thus, by knowing θ and M_1 we can find the angle of shock wave β.

TASK 2

Study the graph in Appendix A and answer the following questions:

a) *If a Mach 3 flow hits an upwards-deflecting shape (like that shown in figure 4) which turns into the flow by 25^0, what is the shockwave angle produced with respect to the original direction of flow?*

b) *If a triangular-shaped object (like the top two shapes of figure 2), with a triangular angle at the apex of 40^0, where placed in a supersonic flow – at what Mach number would the flow detach?*

It should be noted that as the angle of deflection becomes less, so does the strength of the shockwave produced. It is possible therefore to turn a flow very slowly (gradually) by using a gently curved surface – so as to produce such weak-shockwaves that they are practically unnoticeable. In such a scenario the flow is for all intents and purposes isentropic through the turning process.

Once the angles of the shockwave are known, a simple set of equations, given below, give the rest of the parameter relationships:

$$\frac{p_1}{p_0} = 1 + \frac{2\gamma}{\gamma+1}\left(M_0^2 \sin^2\beta - 1\right) \qquad \frac{\rho_1}{\rho_0} = \frac{(\gamma+1)M_0^2\sin^2\beta}{(\gamma-1)M_0^2\sin^2\beta + 2}$$

$$\frac{T_1}{T_0} = \frac{p_1}{p_0}\frac{\rho_0}{\rho_1} \qquad M_1 = \frac{1}{\sin(\beta-\theta)}\sqrt{\frac{1+\frac{\gamma-1}{2}M_0^2\sin^2\beta}{\gamma M_0^2\sin^2\beta - \frac{\gamma-1}{2}}}$$

TASK 3

If the free-stream pressure is 101 kPa in the case stated in task 2a, what would the Mach number and pressure after the shock be (γ = 1.4)?

TOPIC 4 – THE HYPERSONIC LIMIT

At higher speeds (above mach 5) the equations above tend towards fixed values. These are given by:

$$\frac{p_1}{p_0} \approx \frac{2\gamma}{\gamma+1}\left(M_0^2 \sin^2 \beta\right) \qquad \frac{\rho_1}{\rho_0} \approx \frac{(\gamma+1)}{(\gamma-1)} \qquad \frac{T_1}{T_0} \approx \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} M_0^2 \sin^2 \beta$$

$$\frac{\beta}{\theta} \approx \frac{\gamma+1}{2} \approx 1.2\ for\ \gamma = 1.4$$

However it should also be borne in mind that Hypersonic flow is delineated by the onset of chemical reactions within the flow and these alter flow parameters.

TOPIC 5 – EXPANSION FANS

Prandtl-Meyer Expansion fans occur when a supersonic flow encounters a corner which turns away from the flow as shown in figure 6. Unlike a shockwave an expansion-fan is usually an isentropic process.

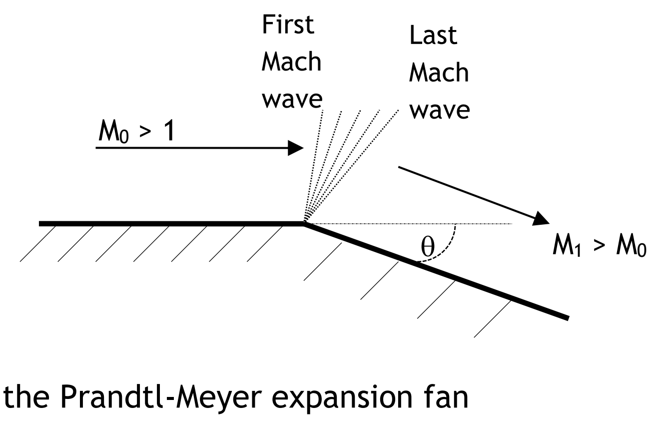


Figure 6, The form of the Prandtl-Meyer expansion fan

The expansion-fan consists of a number of “Mach waves” (pressure waves caused by the fluid parameters changing). The diagram below in figure 7 shows a simplified enlargement of the mach waves shown above.

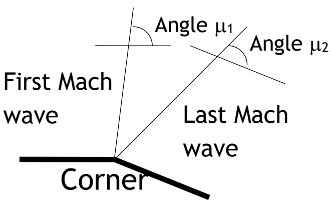


Figure 7, The first and last Mach waves

The first mach-wave makes and angle μ_1 with the flow direction and the last wave makes and angle μ_1.

These angles are: $\mu_1 = \sin^{-1}\left(\frac{1}{M_0}\right)$ and $\mu_2 = \sin^{-1}\left(\frac{1}{M_1}\right)$

The speed after the corner M_1 is calculated using the Prandtl-Meyer function. This in turn uses a calculated angle called the Prandtl-Meyer angle (υ):

The Prandtl-Meyer Angle in region 0 is:

$$\upsilon_0 = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}\left(M_0^2-1\right)} = \tan^{-1}\sqrt{M_0^2-1}$$

And in region 1:

$$\upsilon_1 = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}\left(M_1^2-1\right)} = \tan^{-1}\sqrt{M_1^2-1}$$

Also $\upsilon_1 + \upsilon_0 + \theta$

The idea is to solve these equations to get a value for the speed after the fan - M_1

Unfortunately, like the θ-β-M relationship already discussed this can’t rearranged to provide an explicit solution - so the solution is read of graphs or tables.

We’ll look at a method using a graph of the function as shown in figure 8 and a full useable version is given in appendix B

In this method we first find the Mach number before the turn (M_0 in figure 6) on the x axis and then follow this vertically up until we hit the curve (shown in blue) – we read the angle from the y axis at this point (labelled in θ_0 in figure 8). Next move up the y axis by the angle θ in figure 6 (be careful – some graphs are labelled in radian, others in degrees), this brings us to the angle labelled θ_1 on the graph. From here move across horizontally to meet the curve again. From this point on the curve, move vertically down and read off the final Mach number (M_1 in figure 6) – this is the speed of the flow after the expansion-fan.

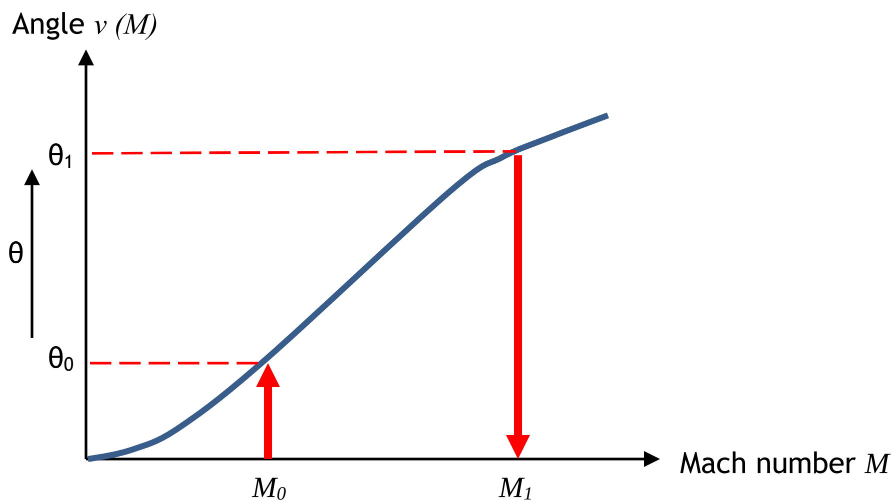


Figure 8, The Prandtle-Meyer function and its use.

TASK 4

From graph in appendix B, if the input Mach number is 5 and turns outward by 15° what is the Mach number after the expansion fan?

The other parameters can now be calculated from the standard isentropic relationships - for reference:

$$\frac{T_1}{T_0} = \frac{1+\frac{\gamma-1}{2}M_0^2}{1+\frac{\gamma-1}{2}M_1^2} \qquad \frac{p_1}{p_0} = \left(\frac{1+\frac{\gamma-1}{2}M_0^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\frac{\gamma}{\gamma-1}} \qquad \frac{\rho_1}{\rho_0} = \left(\frac{1+\frac{\gamma-1}{2}M_0^2}{1+\frac{\gamma-1}{2}M_1^2}\right)^{\frac{1}{\gamma-1}}$$

TASK 5

For the case in task 4 and assuming that the input static pressure is 20 kPa, calculate the down-stream pressure after the turn.

Of course the flow cannot turn through any angle - there is a maximum angle of turn and this is given by:

$$\upsilon_{\max} = \frac{\pi}{2}\left(\sqrt{\frac{\gamma+1}{\gamma-1}} - 1\right)$$ and the maximum turning angle is $\theta_{\max} = \upsilon_{\max} - \upsilon_0$

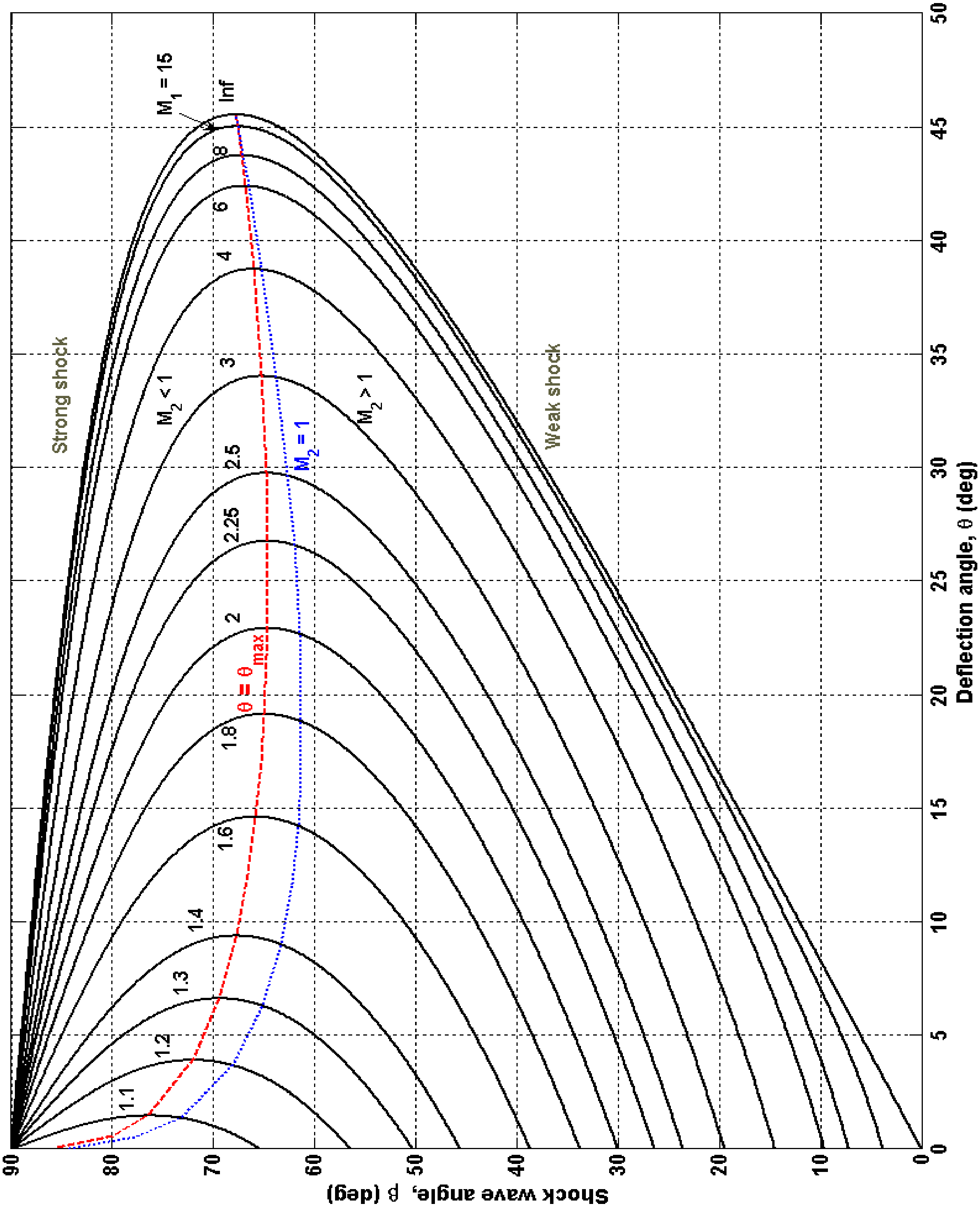
TASK 6

For the case in tasks 4 and 5 calculate the maximum turning angle (γ = 1.4).

SUMMARY

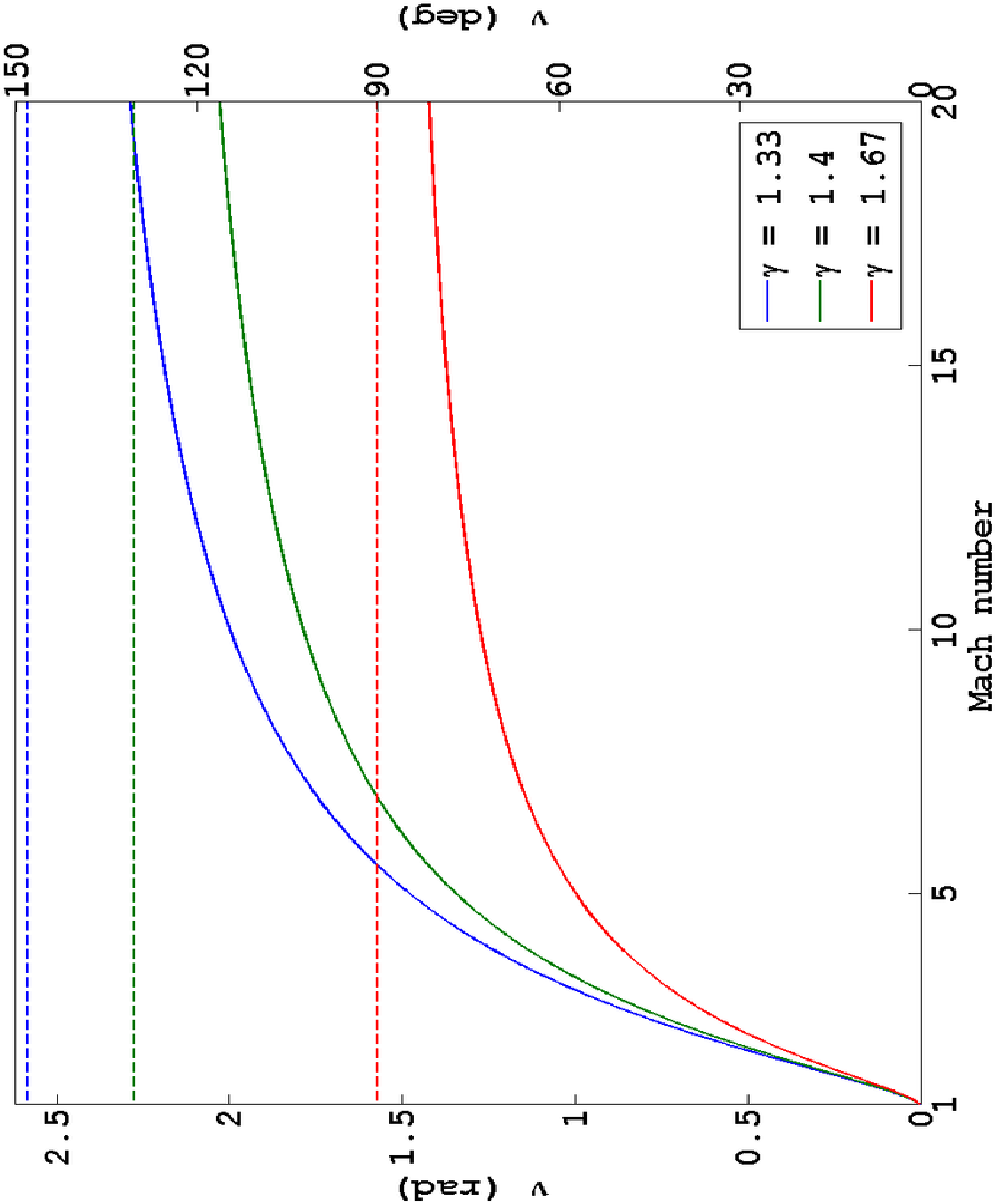
- Shockwaves occur in supersonic flows
- A normal shockwave can occur in a duct or when a shockwave becomes “detached” from a blunt object.
- A normal shock occurs in the throat of a de-Laval nozzle (like a rocket engine).
- The flow on one side of a normal shockwave is subsonic.
- Speed and total pressure fall through a shockwave, but everything else increases.
- The normal shock relations are given on page 5.
- An oblique shockwave occurs when the flow remains attached to a turning surface.
- Oblique shockwaves occur when the surface turns into the flow.
- The shock angle of an oblique shock is given by the θ-β-M relationship or graph.
- The graph also allows us to calculate when the flow becomes detached and/or subsonic.
- The flow after an oblique shock can be supersonic or subsonic.
- The Oblique shock relations are given on page 7.
- At hypersonic speeds the relations tend towards fixed numbers.
- A Prandtl-Meyer expansion fan occurs when a surface turns away from the flow.
- Expansion-fans are isentropic.
- The Mach number after an expansion fan is given by the Prandtl-Meyer function which is often printed as a graph or tables.
- The isentropic expansion-fan relations are given on page 10.
- The maximum turning angle (before detachment) of an expansion fan can also be calculated simply.

APPENDIX A – SHOCKWAVE ANGLE RELATIONSHIPS



(Image “ObliqueShockAngleRelation” by Myth, license: listed as “copyright free, public domain” by author)

APPENDIX B – THE PRANDTL-MEYER FUNCTION



(Image “Prandtle-Meyer Function” by Mythealias, license: listed as “public domain” by author)

REFERENCES AND BIBLIOGRAPHY

You may wish particularly to read the appropriate sections in the “Fundamentals of Aerodynamics” and “modern compressible flow” by Anderson. There are also many YouTube videos and Wikipedia entries illustrating many of the important concepts.

Two useful resources are the Wikipedia pages on “Shockwaves” and “Oblique shocks” and the series of NASA pages on the same topic (which also contain Java simulations) you can find these by searching for “NASA shock wave”.

There are also a number of on-line shockwave calculators and shockwave tables. You can use these to check your results from the tasks above.

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