The scramjet engine

1. Overview

a) What is a Scramjet?

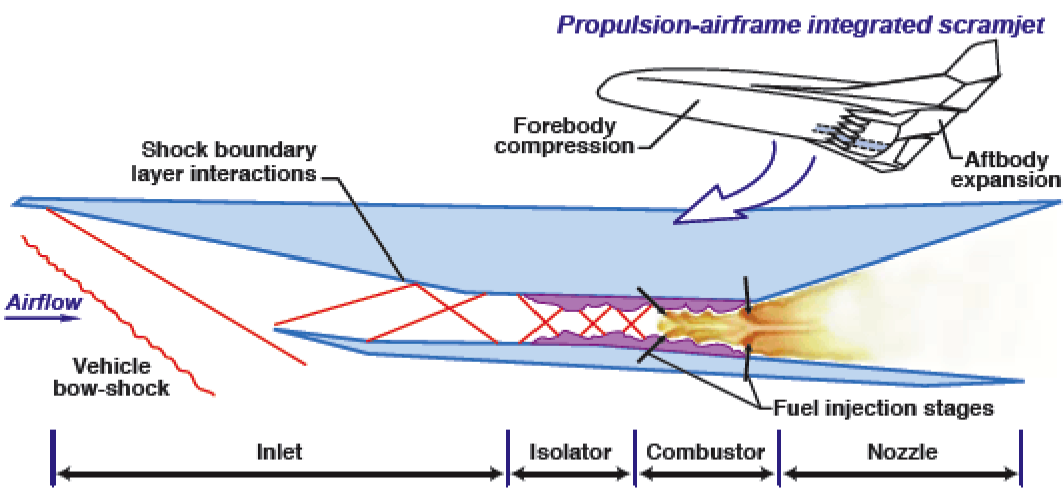
A Scramjet engine is an airbreathing aircraft propulsion device designed to exceed speeds of Mach 5 and could potentially power an aircraft from London to Auckland in 3 hours or even straight into outer space.

The normal jet (gas turbine) engine will work up to speeds approaching the speed of sound (but not exceeding it), and with an appropriate duct before its inlet (to slow the incoming flow to below the speed of sound), will operate at aircraft speeds of up to approximately Mach 3. Above this speed, the forward duct causes too much drag and an unsustainable rise in flow temperature.

Above Mach 3 an alternative engine topology is the Ramjet – this has no moving parts and replies on inherent air-speed to "ram" air into the engine intake (and thereby compress it). The rest of the engine is similar in principle to a gas turbine – fuel is sprayed into the flow and burnt, resulting in thrust. Flow in a ramjet combustor is subsonic. Ramjets are used in some missile systems.

Above Mach 5, the flow into the ramjet engine becomes too hot for the engine materials and for good combustion. The only option is to not slow the stream so much – resulting in a supersonic flow all the way through the engine – this is a Scramjet. The general idea is similar to the ramjet – but fuel must be sprayed and brunt in a supersonic stream. No one has ever got a scramjet to work satisfactory.

A typical scramjet engine is shown below:



Scramjet engine

(origin: NASA, Author: Unknown, Licence: NASA terms and conditions)

A comparison diagram of the engines mentioned above can be found at:

https://en.wikipedia.org/wiki/Scramjet#/media/File:Turbo_ram_scramjet_comparative_diagram.svg

And an explanation of their necessity for space travel:

https://www.youtube.com/watch?v=O50Qnob6A_A

And more in-depth comparison with the other engines:

https://www.youtube.com/watch?v=RwjXbWdbn7A

An "flythrough" animation of a scramjet prototype can be seen here:

https://www.youtube.com/watch?v=fHRwgf4px9w

b) The history of the Scramjet and its current status

A short summery of the scramjet's history and its current development can be found in its Wikipedia entry in the "history" section:

https://en.wikipedia.org/wiki/Scramjet

2. Problems, challenges and issues

There are many issues with developing an engine to operate at hypersonic speeds, these include:

a) Flow transit-time

At high Mach numbers the flow transits through the engine in a few milliseconds and during this time it needs to be compressed, mixed with the fuel, ignited and the fuel fully burnt. This means that the time available for mixing and burning is measured in microseconds.

b) Adding energy to the flow

Because the flow has already slowed down from a high Mach number to a lower one, it is very hot (the kinetic energy in the flow having been converted into heat energy). This means that the combustion process has to add further energy to an already highly energized, hot flow – which is difficult, inefficient and results in low thrust. High temperatures can also degrade the fuel or make it react in undesirable ways (see below).

c) Reacting flows

Because the flow is hot, it may start reacting chemically in undesirable ways – for example by breaking nitrogen and oxygen molecules into their constituent atoms – these may then recombine with each other (forming, for example, NO_x reaction products) or with constituents of the fuel. This could absorb energy and/or produce pollutants.

d) Drag losses

Aerodynamic drag is not a single phenomenon but the result of several acting together to produce an overall retarding effect on an object. Examples are form drag (the result of the bulk shape of the object causing an obstruction to the flow), skin drag (the result of fluid-friction along the object's surface), induced drag (drag which results from lift) and wave drag (caused by shockwaves). Some of these factors increase as the square of velocity and so at high Mach numbers the drag values are enormous and low-drag design is critical. The result is that the engine is finely balanced in terms of drag and thrust.

e) Testing

Very few facilities around the world have practical hypersonic testing facilities and even supersonic wind-tunnels are rare, have a small test diameter and are often pulsed in nature (they produce only a short burst of flow). This means testing new designs is restrictive, time consuming, expensive and difficult. Much emphasis is therefore placed on CFD simulations and few of the resulting designs are proved experimentally.

f) Flame holding

In lower speed airbeathing engines, combustion stability is maintained using a protecting metal or ceramic structure called a "flame holder". This basically ensures that the flame does not blow out. However, projections into the engine cavity would cause too much drag in the case of a scramjet and the situation is complicated by the supersonic nature of the flow (for example it generates shockwaves). The combustion ignition and stability problem is sometimes compared to trying to light a match in a hurricane.

g) Mixing

Before the fuel-air mixture is burnt, it must be thoroughly mixed (this is critical in order to get the maximum energy from the fuel, especially in-light of the problems with hot flows and drag outlined above). However, there is minimal time for this and the supersonic nature of the airflow forms shockwaves between it and the fuel - preventing good mixing.

h) Boundary layers and unstart

High speeds result in thick and viscous boundary layers – these can cause efficiency issues and can even block off (choke) the airflow if they extend too far into the duct – the result of this is the flow stalling and combustion failing – referred to as "unstart".

i) Variable geometry

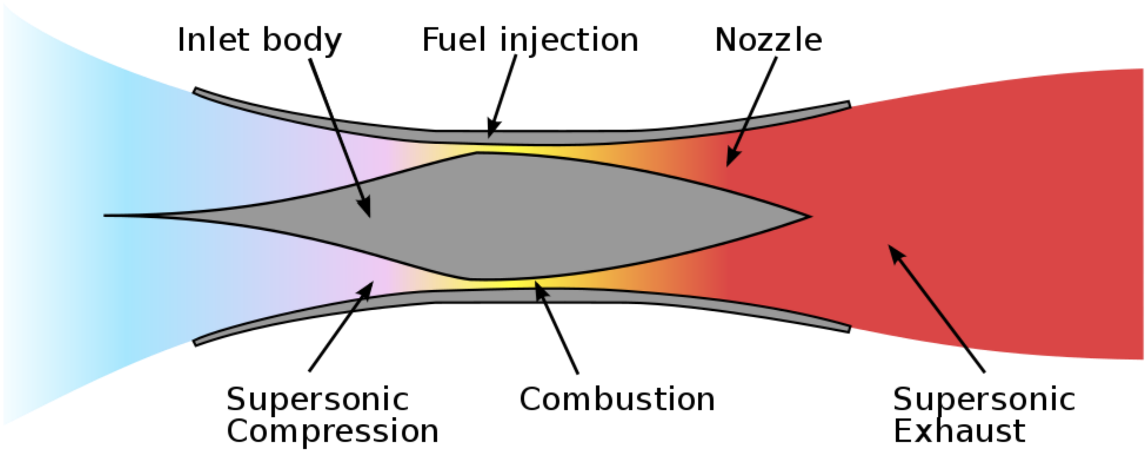
A working scramjet engine would accelerate from ground level, potentially into orbit at Mach 25. This would require the geometry of the engine to change throughout the flight envelop to accommodate the different flow conditions.

Many of the issues outlined above contribute to an over-all efficiency problem for the engine and it is very finally balanced, in the end, between available energy and drag.

3. The scramjet from inlet to exhaust

a) General structure.

The scramjet comes in two basic forms: the axisymmetric form and the linear form. The linear form is shown in the diagram on page 1. The axisymmetric type is shown below:



(origin: Wikipedia, Author: Luke490, Licence: CC BY-SA 3.0)

Both forms have the same basic structure. The purpose of *intake* or *inlet* section is to collect and direct air into engine while ensuring that the resulting flow has the correct pressure, temperature and velocity to ensure good operation. In addition, it must do this as efficiently as possible and avoid several other practical problems like generating stall conditions.

After the intake is usually a short section of, often straight, duct called the *isolator.* The purpose of this is to protect the conditions at the intake from those at the combustor and vice-versa. It also ensures the flow is uniform and may further raise the static pressure. The isolator usually has a series of internal oblique shockwaves (called a "shock-train") which serves this purpose.

Fuel is next introduced into the flow from fuel *injectors* and the resulting mixture burnt in the *combustor*. Achieving a good fuel-air mixture is critical to transferring maximum energy to the flow and this is a huge problem as outlined in section 2 above. Because of the speed of the flow, there are only microseconds for mixing and combustion to happen, and any serious projections into the duct will cause major drag and flow disruption. It is generally considered that mixing and combustion are the major obstacles to making a genuine feasible scramjet.

The nozzle section converts the energetic flow from the combustor into thrust - and works in a similar way to other jet and rocket engine nozzles – except that the flow is supersonic from the outset and therefore only a diverging section is necessary.

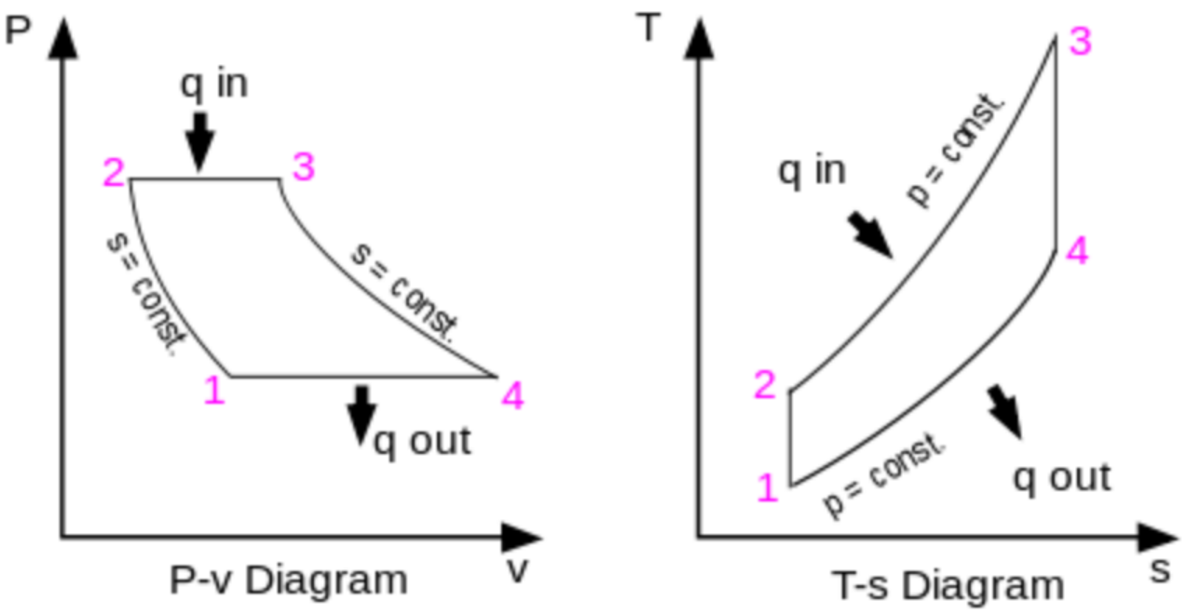
b) The scramjet cycle

Thermodynamically the scramjet conforms (like many other Jet engines) to the general principles of the *Brayton Cycle*:

https://en.wikipedia.org/wiki/Brayton_cycle

The distinguishing attribute of this cycle is that it is a constant pressure process. This is because combustion takes place in an open duct exposed on both sides to a constant pressure reservoir (the input from the isolator on one-side and ambient air on the other).

The Brayton cycle is shown conceptually below:



(origin: Wikipedia, Author: Duk, Licence: CC BY-SA 3.0)

Process 1-2 is the compression at the intake and we can see from the P-V diagram how volume drops as the air is scooped up and pressure rises as it is compressed. Ideally, this is an isentropic (loss-less) process – as shown in the T-S diagram by the constant entropy.

Process 2-3 is combustion. As mentioned above, because the combustor is open, this takes place at constant pressure. Heat is added to the flow increasing its entropy.

The hot gas is next expanded through the exit or exhaust nozzle in process 3-4. Again, to get maximum transfer of heat to thrust, this should be a loss-less process.

Finally process 4 to 1 is the return to the start state of the cycle (at atmospheric temperature and pressure).

As with all cycles, the work done on the engine is the area enclosed by the PV diagram and the ST diagram gives us the heat transfers (the area under process 4-1 being the heat transferred out of the system by the exhaust, under 2-3 being the heat transferred in through combustion and the area enclosed by the cycle being the work transferred by this heat exchange).

The efficiency of the ideal (no loss) cycle can be estimated by comparing it with a Carnot cycle and turns out to be:

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma - 1}{\gamma}}$$

Note the importance of the compressor to the entire cycle. A discussion of the cycle can be seen at:

https://www.youtube.com/watch?v=mstw1qrAGvc

c) Thrust produced

The force produced by an object is given by Newton's second law:

$$F = ma$$

However, in a fluid, mass is a slippery concept – after all the fluid is moving – so what we usually measure is mass-flow rate (in kg/s):

$$\dot{m} = \frac{dm}{dt}$$

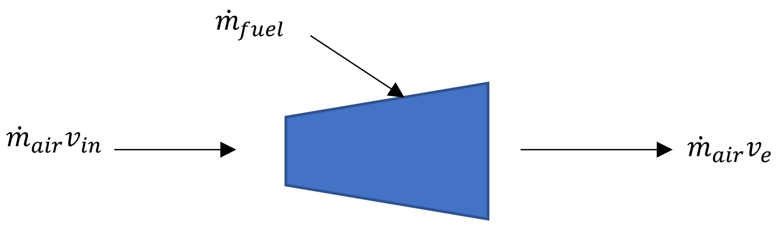
We can use this to manipulate Newton's equation:

$$F = ma = m\frac{dv}{dt} = \frac{dm}{dt}v = \dot{m}v$$

So, the thrust produced by a jet of fluid is:

$$F = \dot{m}v$$

Now, in an engine with an intake, we'd also need to subtract the momentum of the incoming air and add that of the fuel as shown below:



So, the total thrust is:

$$F = (\dot{m}_{air} + \dot{m}_{fuel})v_e - \dot{m}_{air}v_{in}$$

If the exhaust and inlet pressures are not equal, there is also a thrust contribution from excess pressure:

$$F = PA$$

So, this contribution would be:

$$F = (P_e - P_{in})A_e$$

In these equations e = exhaust, and P = pressure.

A discussion of this can be found at:

https://www.youtube.com/watch?v=E0zLVIoZLIg

d) Mission envelope

In order to maintain a constant and predictable flow through the engine (and hence a satisfactory fuel mixture and good combustion), the craft follows a flight path which aims to keep a constant dynamic pressure at the inlet of the engine. The dynamic pressure is given by:

$$p_d = \rho \frac{v^2}{2}$$

We can also write this in terms of other parameters if we note that the speed of sound is given by:

$$a = \sqrt{\gamma RT}$$

And the equation of state is:

$$p = \rho RT$$

Substituting these we get:

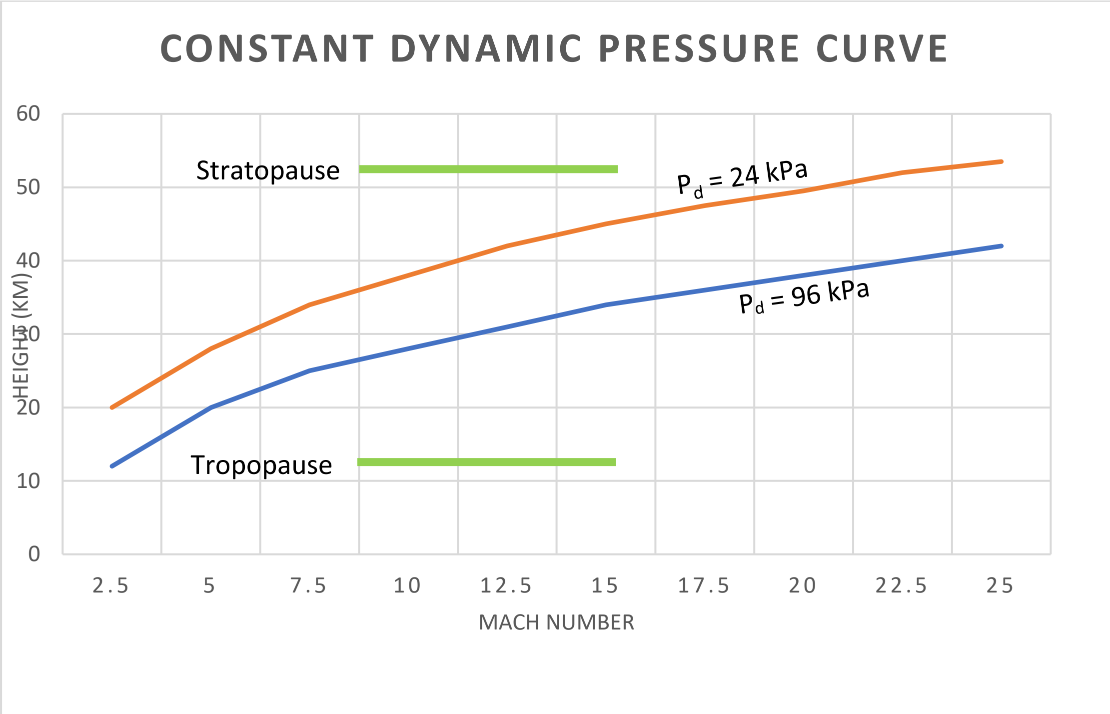
$$p_d = \frac{\gamma p_0 M^2}{2}$$

To maintain good pressures and temperatures in the compressor, the values chosen are usually between 24 kPa and 96 kPa.

For a standard atmosphere:

https://en.wikipedia.org/wiki/International_Standard_Atmosphere

we can calculate a flight envelope which will maintain such a pressure as shown below:



e) Scramjet models

A number of researchers have proposed models for flow through a scramjet engine and used these to predict flow features at various points in the system.

One of the most important criteria used in such models is to ensure that the flow temperature at the entrance to the combustor is sufficiently low, so as to provide good burning conditions (see section 2, b and c above).

The stagnation temperature T_s in a compressible flow is related to the flow temperature T by the following equation:

$$T_s = T\left(1 + \frac{\gamma - 1}{2} M^2\right)$$

So, if we say that the flow temperature at the inlet entry is T_0 and the Mach number is M_0, and the temperature and Mach number at entry to the combustor is T_C and M_C, then we have:

$$T_s = T_0\left(1 + \frac{\gamma - 1}{2} M_0^2\right) = T_C\left(1 + \frac{\gamma - 1}{2} M_C^2\right)$$

We can rearrange this for the Mach Number at the entrance to the combustor:

$$M_C = \sqrt{\frac{2}{\gamma - 1}\left[\frac{T_0}{T_C}\left(1 + \frac{\gamma - 1}{2} M_0^2\right) - 1\right]}$$

In the hypersonic limit this can be reduced to:

$$\frac{M_C}{M_0} = \sqrt{\frac{T_0}{T_C}}$$

If we now put in some values for the maximum burning temperatures for typical fuels (like hydrogen, methane or paraffin) we get an approximate relationship for the maximum Mach number at the combustor inlet:

$$M_C \cong 0.38 M_0$$

Obviously though this does vary a little with design and fuel used.

One of the most famous pioneers and contributors to scramjet development was Frederick S. Billig (1933 - 2006): https://en.wikipedia.org/wiki/Frederick_S._Billig

He proposed a scramjet model (see bibliography for reference) based on such considerations. His calculated values for the freestream and combustor intake parameters are shown below (the combustor entry area is here called the "mixing section" as this is where fuel-air mixing takes place):

Free stream Mach number	Altitude (km)	Mixing section Mach number	Pressure ratio	Mixing section pressure (×10^5 Pa)	Mixing section temp (°C)	Mixing section temp (K)	Mixing section speed (m/s)
3	14.6	1.53	8	1.0	140	413	620
4	17.5	1.95	16	1.28	244	517	879
5	20.0	2.36	25	1.37	339	612	1158
6	22.3	2.77	35	1.35	437	710	1454
7	24.4	3.14	47	1.31	533	806	1755
10	29.1	4.14	90	1.23	815	1088	2665
15	34.8	5.50	186	1.10	1327	1600	4239
20	42.0	6.65	314	0.69	1990	2263	5989
27	54.3	7.69	473	0.22	2609	2882	8292

(Table by author, recalculated from Billig)

Although other researchers have calculated similar data, Billig's original design is still often used as a reference. Such models can be used to simulate the performance of engines and work on practical designs.

f) Efficiency measures

You can find calculations for various engine efficiency measures in the books listed in the bibliography. Some of these include:

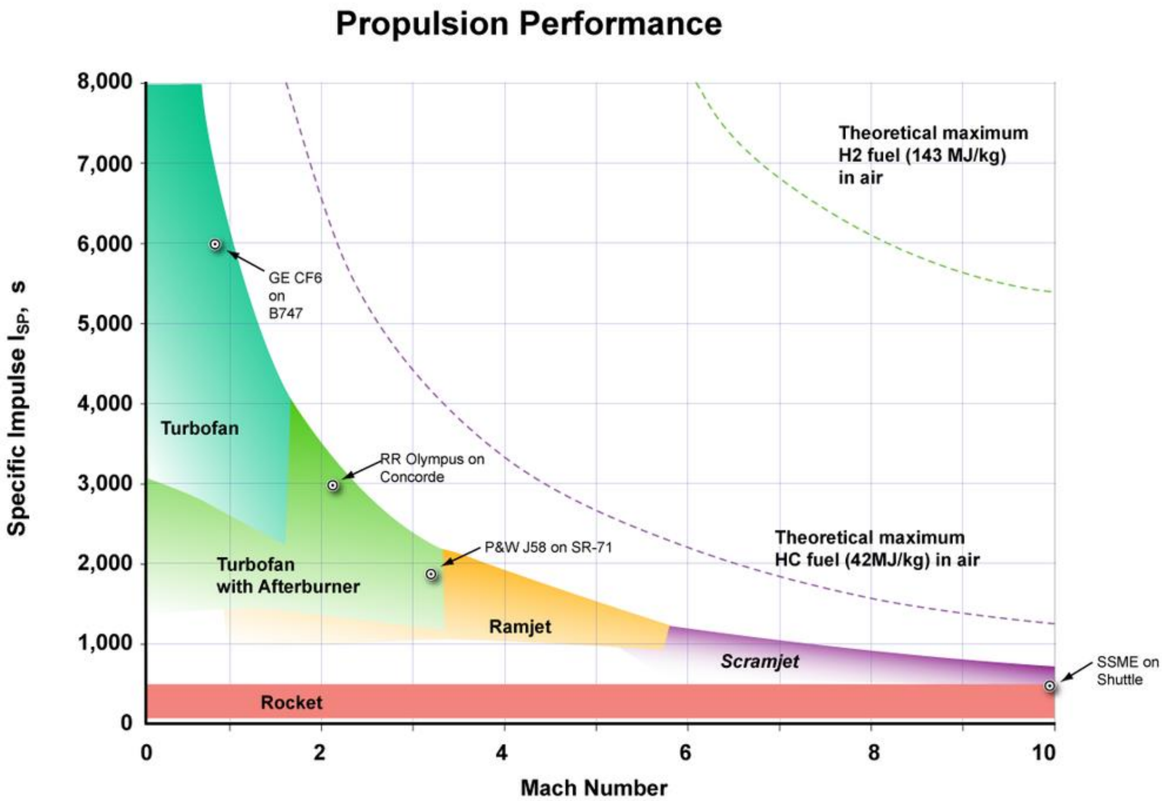
- Thermal efficiency – conversion of heat to thrust (a similar measure is called cycle efficiency).
- Propulsive efficiency – assumes fuel contributes no mass.
- Various component efficiencies – making up these, the intake, isolator, combustor and nozzle all have efficiencies associated with them.
- Overall efficiency – taking all the others into account.

Overall though one of the most common methods of assessing efficiency in aerospace vehicles is the *Specific Impulse* (I_{sp}), this is the impulse (change of momentum) imparted to the vehicle, per unit of fuel used. This turns out, in one formulation, to be equal to the trust produced divided by the mass flow rate of propellent:

$$I_{sp} = \frac{F}{\dot{m}}$$

Note however, that there are other ways of defining this figure (and you can derive other equations by combining it with the thrust expressions, for example).

Shown below is the calculated and measured, practical and theoretical specific impulses for various engines (in this formulation the units of I_{sp} are in seconds):



(origin: Wikipedia, Author: Kashkhan, Licence: CC BY-SA 3.0)

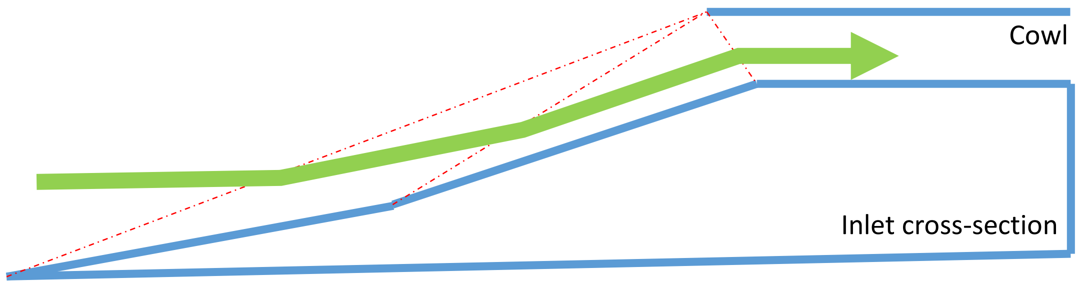
4. The inlet and isolator

a) General design, purpose and options

It was mentioned above that the purpose of the intake is to collect and direct air into engine at the correct pressure, temperature and velocity and do this as efficiently as possible, while avoiding stall and similar conditions. This is quite a tall order!

The intake achieves this using a series of shockwaves. If you are not familiar with shockwave mechanics, you should now pause and read the notes on these.

Each time the air passes through a shockwave, the velocity falls and the pressure rises (which is good – this is what you want from a compressor). However, unfortunately the temperature also rises and the total pressure falls (because a shockwave is not isentropic). A simple shockwave compression system is shown below:

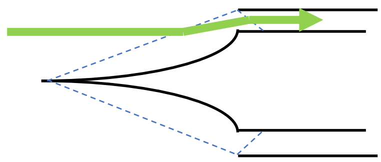


The red dotted lines are the shockwaves, the blue lines are a cross-section of the inlet (which in this case is a linear inlet, like that shown in the diagram on page 1 - but this representation is up-side down compared to that diagram). The green arrow is the airflow.

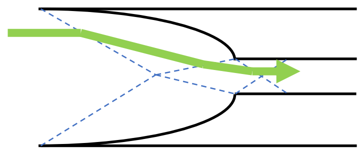
You can note several things about this arrangement:

- The shockwaves redirect the flow (so that it's parallel with the slope of the structure).
- The topology is designed so that the shockwaves terminate on the edge of the cowl – this is typical.
- The last shockwave is internal and redirects the flow along the duct towards the isolator section.

Shown in the diagram over-leaf are a couple of the other commonly discussed designs.



Axisymetrical external compression intake



Axisymetrical internal compression intake

The axisymetrical engine designs have fallen out of favour in recent years because of the low drag configuration of fully airframe-integrated linear types. The internal compression system has several advantages including a shortened length - but is difficult to design and optimise because of the complexity of the internal shockwave pattern. Many other designs have also been proposed over the years, including ones with both horizontal and vertical compression components (sometimes referred to as side-wall compression).

A review of inlets is given here:

https://www.youtube.com/watch?v=XbT-Gz-KNRA

b) Performance

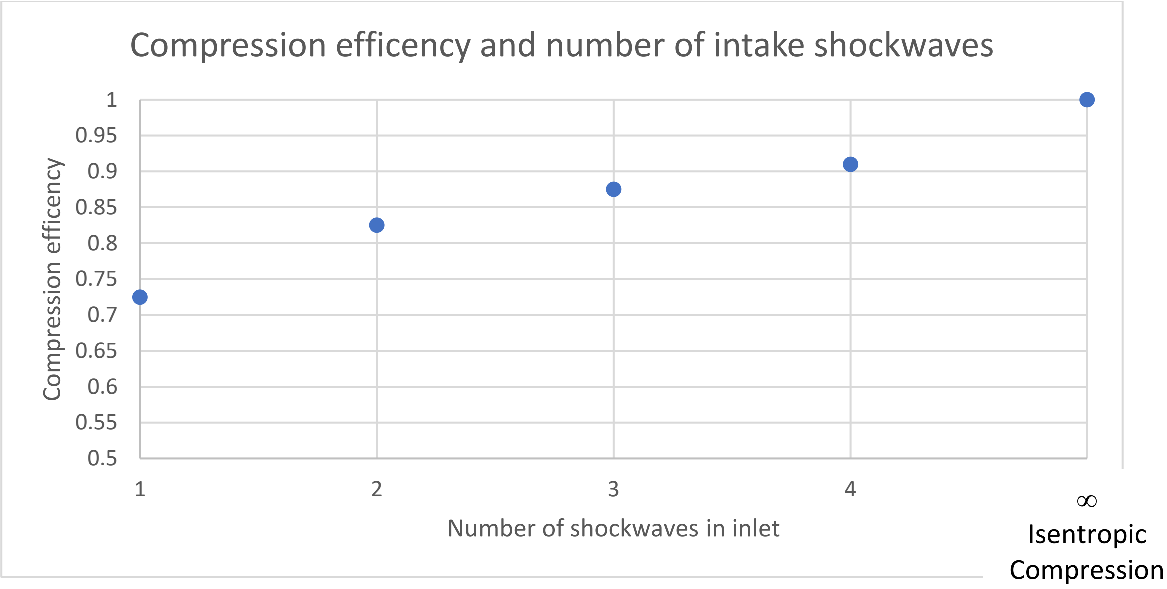
Because the engine is very finely balanced in terms of drag and thrust and as much of the chemical energy as possible must be converted into kinetic, the efficiency of the compression system turns out to be a critical factor in overall engine performance (see the equation on page 6). The compression efficiency in terms of the enthalpies and temperatures shown on the graph on page 5 is:

$$\eta_c = \frac{h_2 - h_1}{h_{2x} - h_1} = \frac{C_p(T_2 - T_1)}{C_p(T_{2x} - T_1)} = \frac{T_2 - T_1}{T_{2x} - T_1}$$

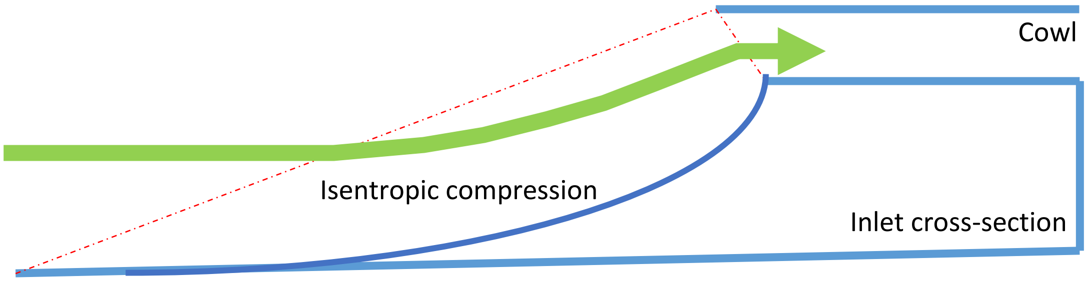
Where the numbered subscripts correspond to the isentropic values at those points and x is the actual value (the efficiencies of the other engine components can be similarly calculated).

Unfortunately, shockwaves are not isentropic – and therefore have losses associated with them. These losses manifest themselves as a decrease in total pressure across the shock.

However, it turns out that it is possible to decrease losses: Suppose we took the inlet shown on page 11 and instead of changing the path of the flow with three shocks as shown (two external and one internal), we added further steps or ramps to the front – this would introduce more shockwaves, but their effect would be weaker as they turn the flow less each time. Each of these weaker shocks is closer to an isentropic process. In the limiting case of this there are an infinite number of shockwaves, each infinitely weak – and this corresponds to an isentropic compression. The effect of this is shown in the graph overleaf.



This limiting case is known as an *Isentropic Compression Surface* and is a gently curved surface which compresses the flow gradually. Unfortunately, detailed design of this surface is somewhat involved, and we don't have time to cover it in here.

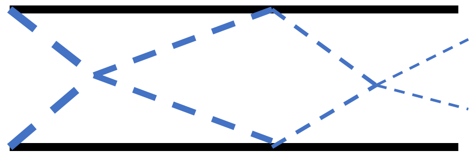


If you are unfamiliar with isentropic compressible flow, please read the separate notes on this now.

c) Other inlet issues

There are two further important issues which we should address before leaving the topic of the intake. The first of these is the isolator. This is normally a fairly straight section of duct, which connects the combustion section to the intake. Its purpose, as mentioned previously, is to isolate pressure changes in either of these from affecting the other. Usually however, it is the intake which needs protecting from the combustion section, as changes of pressure there could cause the inlet to stall.

A normal shockwave is one way to achieve isolation - since the shock isolates parameters upstream of the shock from those downstream. Of course, we cannot use this mechanism because it infers a supersonic to subsonic transition. However, it can be shown that a string of oblique shocks can act in a very similar way. Such a string is known as a *shock train* and will also serve to compress the flow further. The diagram below illustrates the idea.



Shocks reflect from side-walls and each other as they proceed down the duct. They also gradually become weaker.

Finally, although we have discussed the design of an inlet at a single speed, of course across the flight envelope the conditions change as shown in the diagram on page 8. In a practical engine this would require the use of a variable geometry intake which can alter its physical configuration through the operational range. This is common practice on high-speed aircraft like Concord and the Lockheed SR71 and you might like to give some thought as to how it might be facilitated.

5. Mixing and combustion

a) General problem

We have already discussed how, at high Mach numbers, drag values are very large and it is difficult to add further kinetic energy to an already energised air-stream. As we now know, this means that the engine is finely balanced in terms of its thrust and drag components and a low-drag performance is essential for success. It may also be understood from this that good conversion of the fuel's chemical energy into thrust is essential.

Yet, at high Mach, air passes through the engine in around a millisecond, meaning that the fuel must mix with the air, burn and release its energy in a few tens of microseconds. To achieve maximum extraction of energy, the fuel must be mixed stoichiometrically at the molecular level, during this time. This must be done in such a way that it does not disrupt the flow enough to cause an unacceptable increase in drag. The mixture must then be burnt, but without the aid of the flame-holding structures used at lower speeds - as projections into the duct would cause form-drag. Finally, all this must be done without disrupting the conditions at the inlet.

b) The importance of diffusion

Only diffusion can provide the necessary microscopic mixing across the fuel-air boundary and achieve a stoichiometric mixture *at the molecular level*. Unforced diffusion is controlled by *Fick's Law*, which in this case (in one dimension) may be written as:

$$J = -D_{FA} \frac{\partial C}{\partial y}$$

Here D_{FA} is the diffusion coefficient or diffusivity of the fuel into the air (or vice-versa) measured in m^2s^{-1}, y is distance in m and C is the concentration of the air (C_A) or fuel (C_F) - depending on which one is being measured, usually in (mols)m^{-3}. In this case, the result J is the flux of substance diffusing, in (mols)$m^{-2}s^{-1}$. You may recognise this equation as the steady-state Fourier equation which also describes heat transfer (and several other physical processes too).

Finding values of measured diffusivity of (say) hydrogen into air at the pressures and temperatures of a typical scramjet engine in the published literature is almost impossible. Heiser et al (see bibliography), in their calculations, use the dynamic viscosity μ to obtain a value for diffusivity:

$$D_{FA} = \frac{\mu}{S_C \rho}$$

Where ρ is the density in Kgm^{-3} and S_C is the Schmidt number, μ in Nsm^{-2} is approximately given for air by:

$$\mu = 1.46 \times 10^{-6} \frac{T^{\frac{3}{2}}}{T + 111}$$

The weakness of this approach is that it assumes a constant value for S_c - which is known to vary. Nether the less, by assuming a value of $S_c \approx 0.2$ - a typical measured value of hydrogen in air (for other fuels, typically $S_c \approx 1$), useful results can be obtained as illustrated below.

An alternative approach is to derive an expression for D_{FA} directly from kinetic theory. One such formula in SI units is:

$$D_{FA} = \frac{3}{8nd_{FA}^2} \sqrt{\left(\frac{kT(m_F + m_A)}{2\pi m_F m_A} \right)}$$

Where n the number density of molecules, k is Boltzmann's constant, T is absolute temperature, m_F and m_A are the masses of the fuel and air molecules (obviously an average value for air) and d_{FA} is the average diameter of a molecule in the system.

Putting in the various constants for hydrogen and air, this equation reduces to:

$$D_{FA} = \frac{5.63 \times 10^{18} \sqrt{(665 \times T)}}{n}$$

Again, n may be calculated from kinetic theory and all parameters are in SI units.

The disadvantage of this method is that the typical assumptions of Kinetic Theory are applied (for example, assuming that gasses are perfect and molecules are spherical).

Calculated values from both these methods are tabulated in the table below, for the parameters given at the injectors, just before combustion, in Billig's engine.

Free-stream Mach Nº	Temp (K)	ρ (kg/m^3)	μ_{air} (Ns/m^2)	D_{FA} (cm^2/s) S_c = 1, note 1	D_{FA} (cm^2/s) S_C = 0.2, note 2	n (#/m^3) × 10^{24}	D_{FA} (cm^2/s) note 3
5	700	1.24	3.33	0.27	1.35	27.3	1.41
7	810	0.563	3.65	0.65	3.25	12.4	3.32
10	1090	0.39	4.37	1.12	5.6	8.65	5.53
15	1600	0.238	5.46	2.3	11.5	5.25	13.02
20	2260	0.105	6.62	6.3	21.5	2.33	29.64

Note 1: Values calculated by Heiser and Pratt's method for most fuels. Values are given in cm^2/s for convenience, to convert to m^2/s divide by 10000.

Note 2: Values calculated by Heiser and Pratt's method for hydrogen and air.

Note 3: Values calculated from Kinetic Theory for hydrogen and air.

The values calculated by continuum and molecular methods in this case differ by less than 4.2% up to Mach 15 and then diverge to a maximum of 27.7% difference in extreme conditions. The accuracy of these values may also be compared against the few available measured figures at similar gas parameters in the literature. In the case of the Kinetic Theory calculations, this differs by less than 1.5% and by around 30% in the case of the continuum calculation.

c) Practical calculation

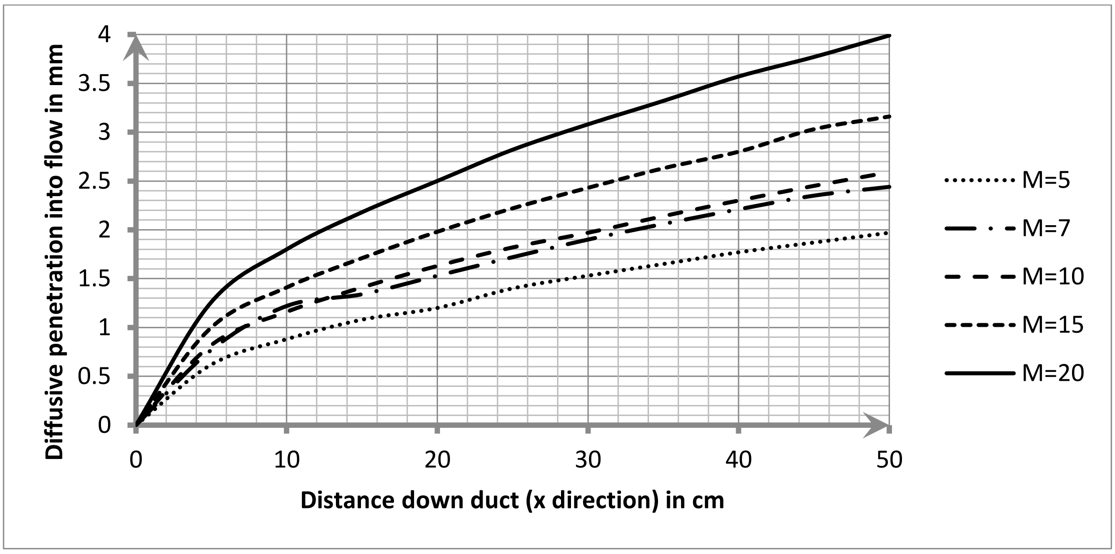
To calculate the penetration of the fuel into the air stream by diffusion, there are several roughly equivalent methods given in various references. A common approach is to use the formulae in Heiser et al. They give the approximate thickness of the mixing layer δ as:

$$\delta \approx 8\sqrt{\frac{D_{FA}x}{u}} = 8\sqrt{D_{FA}t}$$

Where u is the convective velocity, in this case the velocity of the stream, assuming both fuel and air are moving in the same direction together. The axial distance down the duct is x and t is the time interval being considered.

d) Turbulent mixing and injectors

The graph below shows the result of plotting the equation above – the diffusive penetration of fuel into the airstream verses the distance along the duct which the flow has travelled at various axial (free-steam) velocities.



As can be seen, by the time the flow moves down the duct by, for example 25cm, the penetration is only a few millimetres.

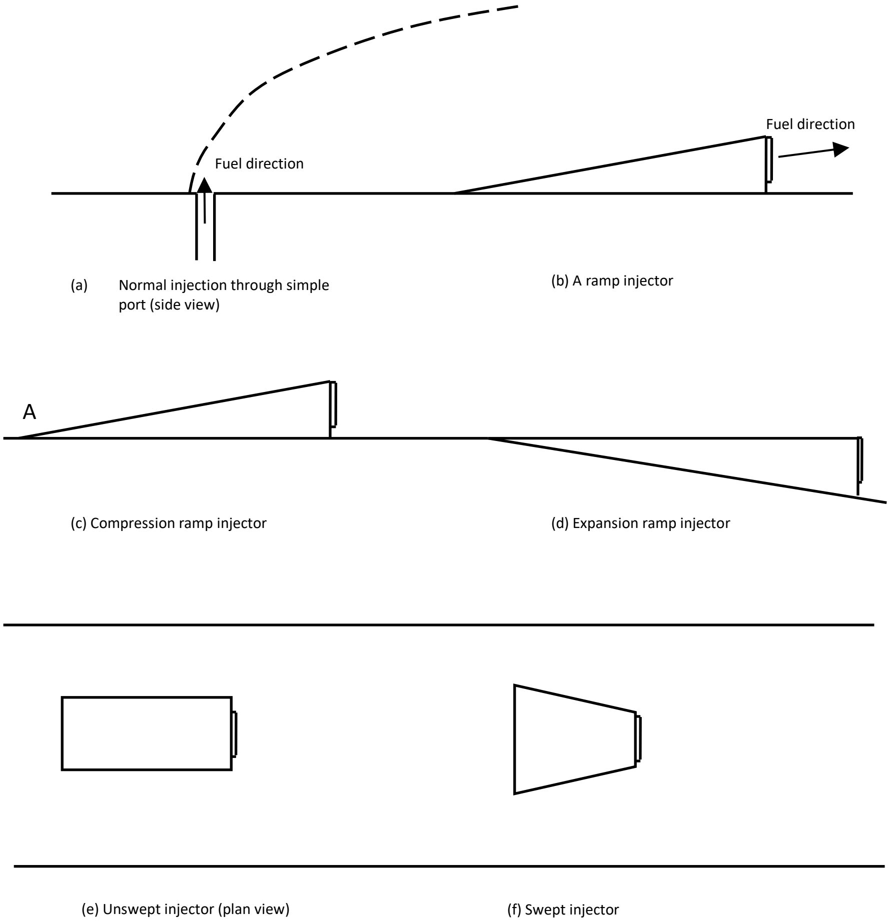
The importance of these figures is this: Whatever type of macro-mixing is used to bring the fuel into close contact with the flow (injectors, vortex-generators, struts, pylons, etc.), it must result in the fuel and air being *macroscopically* mixed to within the distances shown in the graph, as only diffusion can "finish the job" and ensure mixing at the molecular level.

Some videos which will help you understand mixing can be found here:

https://www.youtube.com/watch?v=KC17U4fzEMw

https://www.youtube.com/watch?v=Go9MlukoMro

One way to tackle the problem is to induce turbulent mixing. This enfolds the fuel in the air and provides a large boundary for it to diffuse across. The fuel is injected in via an injector and the whole contraption induces turbulent mixing. Some of the more common injector topologies are shown overleaf.



However, in this discussion, we've neglected another issue - and that is the effect of compressibility on the problem. Where the fuel and air streams meet, a discontinuity forms. If the streams are relatively supersonic, this takes the form of a shockwave. The shock is an area of high energy and density and effectively a barrier to penetration and therefore diffusion. Even before shockwaves form, a region of increased compression exists, which has a similar effect. To quantify the amount of compression at the boundary between the flows, many authors define a relative speed for the flow components (essentially shifting the frame of reference from the laboratory to that of the free flow). This is often termed the convective Mach number M_c. A common definition for two flows is:

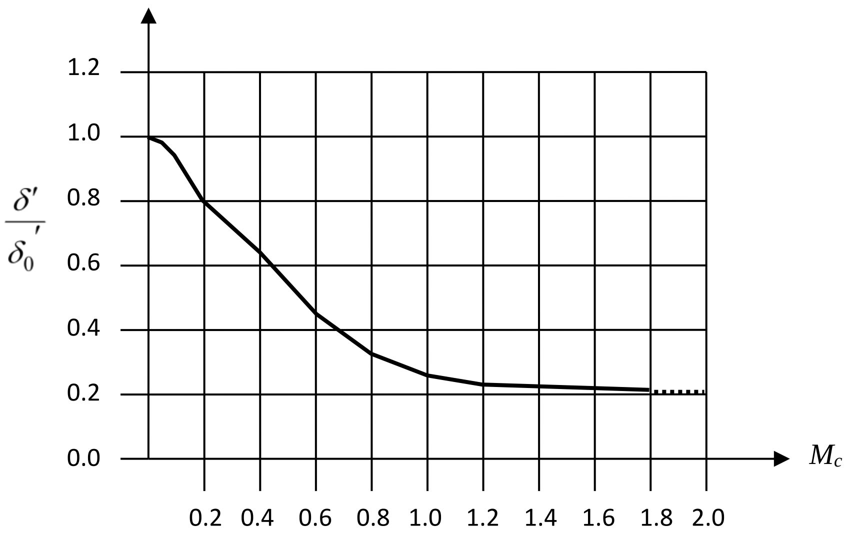
$$M_c = \frac{u_1 - u_2}{a_1 + a_2}$$

Where u_1 and u_2 are the speeds of the flows under consideration and a_1 and a_2 are the speed of sound in these flows. The graph overleaf shows the effect of compression.

In this graph, the spatial rate of increase of the shear layer thickness is labelled δ' and the growth rate of the layer at M_c = 0 is labelled δ'_0 (sometimes called the incompressible growth rate):

$$\delta' = \left.\frac{d\delta}{dy}\right|_{\delta = f(M_c)} \text{ and } \delta'_0 = \left.\frac{d\delta}{dy}\right|_{M_c = 0}$$

So, the term δ'/δ'_0 is the rate of change of the mixing layer, normalised to the incompressible case.



It can be seen from the graph that the amount of mixing decreases rapidly with increasing Mc and the distances calculated from the diffusive mixing values should be adjusted downwards accordingly. The figure tends asymptotically to a value of around $\delta'/\delta'_0 \approx 0.2$, particularly for $M_c > 1$. Therefore, for maximum mixing rate, the velocities of fuel and air should match and, in the worst case, the mixing layer growth is only one fifth of its maximum possible value. A curve fit to the graph gives:

$$f(M_c) = 0.2 + 0.8e^{-3M_C^2}$$

The upshot of all this is clear, although unpalatable. In a practical Scram system, for good air-fuel mixing, even under optimal conditions, the fuel injection system must ensure that the air and fuel are in macroscopic contact within a few millimetres to allow molecular diffusion to take place before combustion.

And here's the issue – this seems difficult in the extreme with current systems – and so no scramjet has ever worked efficiently and it is arguable whether any has ever produced measurable thrust.

Some videos on this topic can be found here:

https://www.youtube.com/watch?v=0zu_2TzvsM0

https://www.youtube.com/watch?v=ywzGY4Wf_8g

e) Combustion

Let's finally now turn to combustion.

For a general hydrocarbon C_xH_y and ignoring non-ideal reactions, the stoichiometric equation is:

$$C_xH_y + \left(x+\frac{y}{4}\right)\left(O_2+\frac{79}{21}N_2\right) \longrightarrow xCO_2 + \frac{y}{x}H_2O + \frac{79}{21}\left(x+\frac{y}{4}\right)N_2$$

In this equation the nitrogen is just considered an inert gas (of course in reality it may react to form polluting compounds with oxygen or fuel components).

For ideal hydrogen combustion (ignoring the nitrogen this time), we just have:

$$2H_2 + O_2 \longrightarrow 2H_2O$$

Since the molecular weight of H_2 is 2 and 32 for O_2, for ideal hydrogen combustion the required mass of oxygen is 8 times that of hydrogen.

For our hydrocarbon fuel, the molar or volumetric ratio fuel to air ratio (mol_F / mol_A = R) is:

$$R = \frac{1}{\left(x+\frac{y}{4}\right)\left(1+\frac{79}{21}\right)} = \frac{84}{100(4x+y)}$$

And in terms of mass:

$$R_m = \frac{36x+3y}{103(4x+y)}$$

In many other internal combustion engines, the mixture is often fuel-rich to ensure good burning (so for example, in a hydrogen system, the O_2 mass ratio is often closer to 4 than 8).

f) Flame holding

An essential part of normal jet engines is the flame holder. This is an obstruction in the flow which generates an eddy and allows the fuel to be burn without "blowing out". These structures are often "V" shaped struts. However, such structures cause drag and so their use in a scramjet engine is difficult.

This is especially tricky because, due to the speed of the flow, the flame is even more likely to extinguish.

The flame holding aspect of scramjet propulsion is not fully solved yet.

6. The exhaust or nozzle section

In many ways the exhaust nozzle is the best understood part of the engine. In fact, it's simpler in some ways than a rocket nozzle because the flow is always supersonic - and so only needs to be divergent to maximise exit velocity.

However, in this section we'll review both convergent and divergent nozzles in the form of a rocket engine – this is useful as it illustrates and further develops important isentropic flow features and details a system that has much wider application than just scramjets and rockets (for example, supersonic wind-tunnels).

a) Review

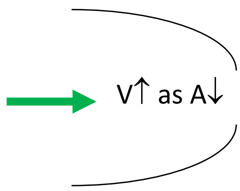
Flow behaviour through converging / diverging ducts is given by the *area-velocity* relationship:

$$\frac{dA}{A} = (M^2 - 1)\frac{dV}{V} \qquad \text{(eqn 0)}$$

This infers that for subsonic flow, that is $M < 1$ (and hence $M^2 - 1 < 0$):

V increases as A decreases

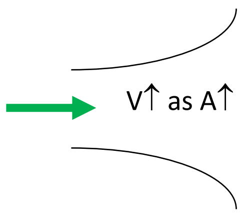
In other words, for speeds less than sound, flow speeds up through a converging duct:



Conversely for supersonic flow, that is $M > 1$ (and hence $M^2 - 1 > 0$):

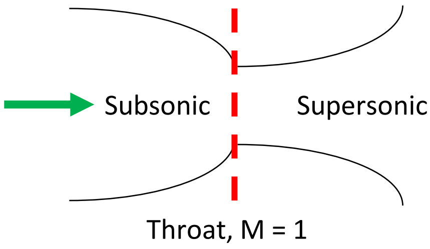
V increases as A increases

In other words, for speeds greater than sound, flow speeds up through a diverging duct:



Therefore, the area–velocity relationship tells us that subsonic and supersonic flow behave in exactly the opposite way (with regard to velocity) in ducts.

So, to speed flow up though a range of speeds from subsonic to supersonic we need a *converging-diverging* duct, also known as a *de Laval nozzle* (after the inventor that suggested it):

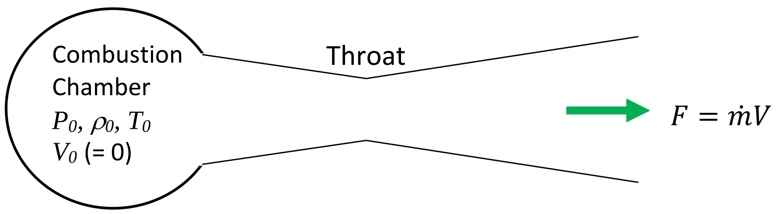


Where the constriction in the centre is called the throat and must have a Mach number of 1 if the flow is to continue to accelerate (and a normal shockwave must therefore be present here).

Since to obtain maximum thrust we want maximum velocity:

$$F = \dot{m}V$$

this is the basis of the rocket engine:



Here the subscript 0 denotes stagnation (stationary) values

The flow through almost all of the engine is isentropic as no shockwaves are present in the converging or diverging parts (in normal operation). Also, because the normal shockwave is approached gently (the speed slowly builds up to it) and similarly exited gently on the diverging side, we can even approximate this isentropically.

b) Further isentropic flow relationships

We are familiar with the standard isentropic relationships (and other applicable gas relationships) and these all apply to this flow:

Energy Balance equation
$$C_p T + \frac{V^2}{2} = const$$

Pressure, density and temperature relationships
$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{(\gamma-1)}}$$

Continuity equation
$$\rho AV = const$$

Equation of state
$$p = \rho RT$$

But it's also possible to write these terms down in terms of the flow Mach number:

$$C_p T_1 + \frac{V_1^2}{2} = C_p T_2 + \frac{V_2^2}{2} = C_p T_0$$

Ignore position 2 and divide through by T_1 and C_p:

$$\frac{T_0}{T_1} = 1 + \frac{V^2}{2 C_p T_1}$$

Now use the relationship for specific heat $C_p = \gamma R/\gamma - 1$ to substitute:

$$\frac{T_0}{T_1} = 1 + \frac{V^2}{2 T_1 \frac{\gamma R}{\gamma - 1}}$$

And finally note that $a = (\gamma RT)^{0.5}$ and $M = V/a$:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

And using the standard isentropic relationships:

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{(\gamma-1)}}$$

We can get the other ratios:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \qquad \text{(eqn 1)}$$

$$\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} \qquad \text{(eqn 2)}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}} \qquad \text{(eqn 3)}$$

Note also that the values of all these ratios can be found at the throat of a rocket nozzle in normal operation by making $M = 1$.

We can also find how mass-flowrate varies with duct area by:

$$\dot{m} = \rho A V$$

$$\therefore \frac{\dot{m}}{A} = \rho V$$

But using the equation of state $\rho = p/RT$ we get:

$$\frac{\dot{m}}{A} = \frac{p}{RT}V$$

And expressing V as a Mach number with $a = (\gamma RT)^{0.5}$ we have:

$$\frac{\dot{m}}{A} = \frac{p}{RT}M\sqrt{\gamma RT}$$

We can not substitute equation 1 in for T and 2 for p giving:

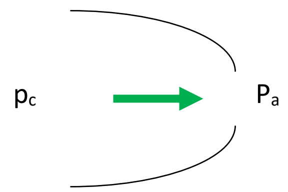
$$\frac{\dot{m}}{A} = \frac{p_0 M}{\sqrt{RT}}\frac{\sqrt{\gamma}}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \qquad \text{Eqn 4}$$

From which we can also simply obtain the velocity (divide by ρ) or mass flow rate (multiply by A).

c) Converging section

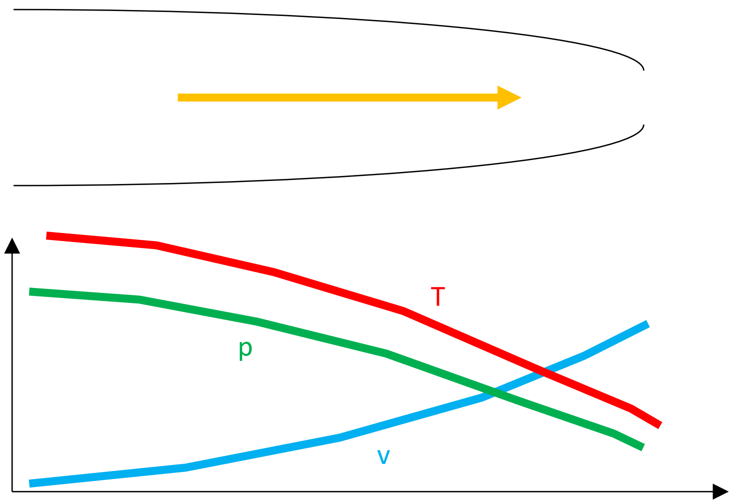
Let's examine what happens in the converging section of duct.

Firstly, obviously, the flow will increase in velocity as it is subsonic. The flow itself is driven by the pressure difference between input and output as shown:



In the rocket of course p_c would be combustion-chamber pressure and p_a would be the ambient (air) pressure at the exit.

If the pressure difference ($\Delta p = p_c - p_a$) were zero, then they'd be no flow. The flow rate then would increase with pressure difference. We can see this relationship from the graph:



Eventually, if we increase the pressure difference enough, the Mach number at the throat will reach one (M = 1) and we can't go to any higher velocity - because, above Mach 1 speed decreases in a converging duct (eqn 0). The pressure ratio necessary to give this condition is given by setting M = 1 in equation 2 and the mass flow-rate is given by setting M = 1 in equation 4.

You can combine these equations with a bit maths and the ideal gas law to arrive at an equation for the required area of the throat for sonic flow:

$$A_t = \frac{\dot{m}}{p_t}\sqrt{\frac{R^* T_t}{\gamma m}}$$

In this equation the subscript t denotes values at the throat, R^* is the universal gas constant (8214 Da m^2 s^{-2} K^{-1} as opposed to the specific constant R, used in the other equations) and m is the molecular mass of the exhaust gas (H_2 = 2 for example).

Two important points should be noted: a) Because the speed cannot be further increased, neither can the flow-rate and we say that the nozzle is choked. b) Once the throat is at M = 1, nothing happening beyond the nozzle can affect the flow up to this point (because no effect can travel backwards through the flow at greater than the speed of sound).

We can also write an equation to describe how the area restriction and Mach number relates to the throat area. Since the mass-flow rates have to be the same all the way through the nozzle:

$$\rho A V = \rho_t A_t V_t$$

$$\frac{A}{A_t} = \frac{\rho_t V_t}{\rho V}$$

Now, adding some creative ratios which multiply out to one unit, we can write:

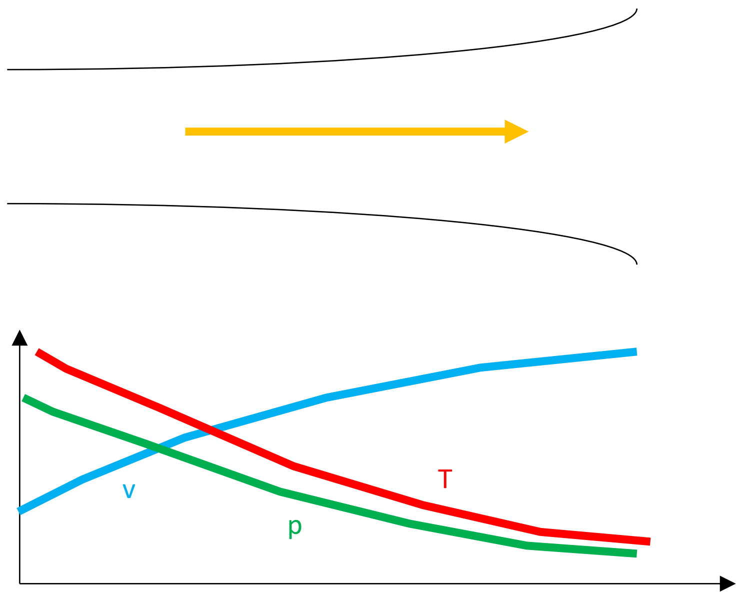
$$\frac{A}{A_t} = \frac{\rho_t}{\rho_0} \cdot \frac{\rho_0}{\rho} \cdot \frac{V_t(a)}{a} \cdot \frac{a}{V}$$

And substituting equations 1 to 3 in, we get:

$$\frac{A}{A_t} = \frac{1}{M}\left(\frac{1 + \frac{\gamma - 1}{2}M^2}{\frac{\gamma + 1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

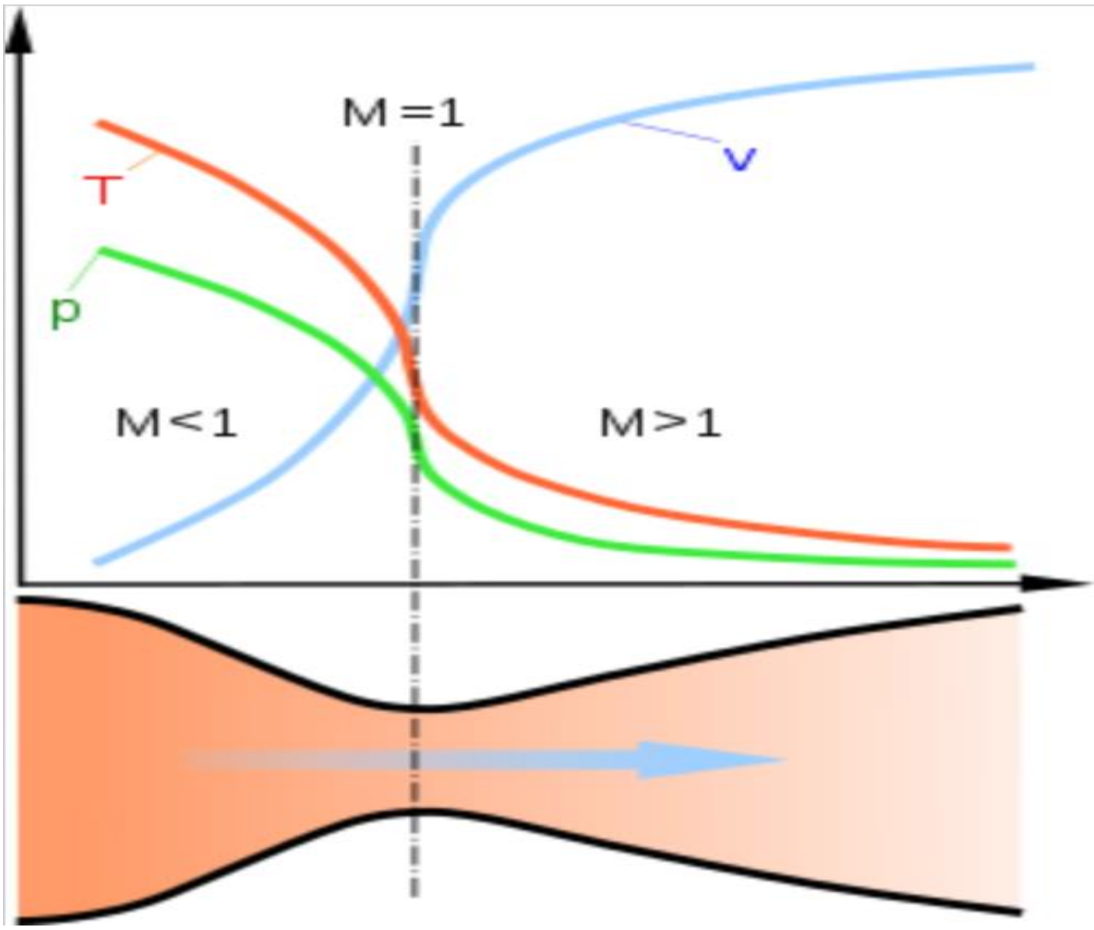
d) Diverging section

Providing that the pressure difference is great enough to sustain flow beyond the throat, it will continue to accelerate as shown in the diagram below:



e) The combined nozzle system

The whole system, for a correctly working nozzle, is shown below:



(Image: “Nozzle de-Laval” by IOK. Origin: Wikipedia, license: listed as “copyright free, public domain” by author)

This perfect flow condition occurs when the nozzle is designed so that the exit pressure is exactly equal to ambient pressure. It’s not generally possible to do this through a rocket flight envelop without a variable topology nozzle. However, this is the nozzle at its most efficient.

The flow exit velocity will be given as:

$$v_e = \sqrt{2C_pT_c\left(1-\left(\frac{p_e}{p_c}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

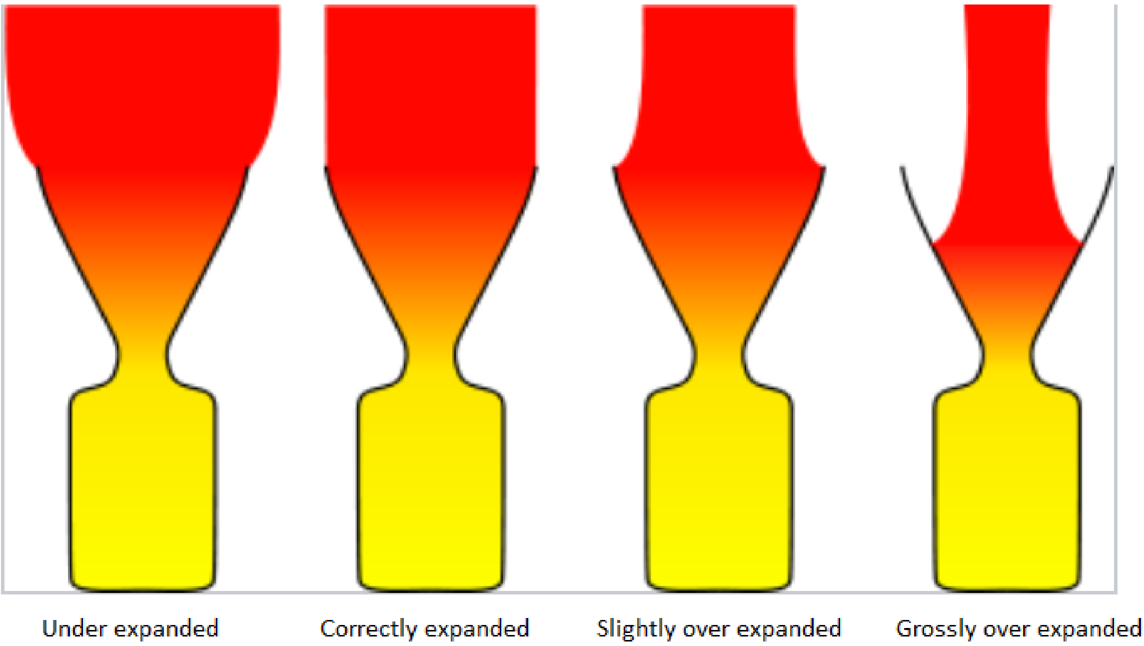
There are three other circumstances however to take note of:

- If the pressure difference is not enough to maintain flow to the end of the diverging section, the flow will revert back to subsonic with a normal shockwave within the nozzle.

As already stated, the ideal situation for maximum efficiency is that that the pressure at the end of the nozzle (p_e) drops to exactly to the ambient, p_a (by good design of the diverging section). If this is not the case the nozzle may be:

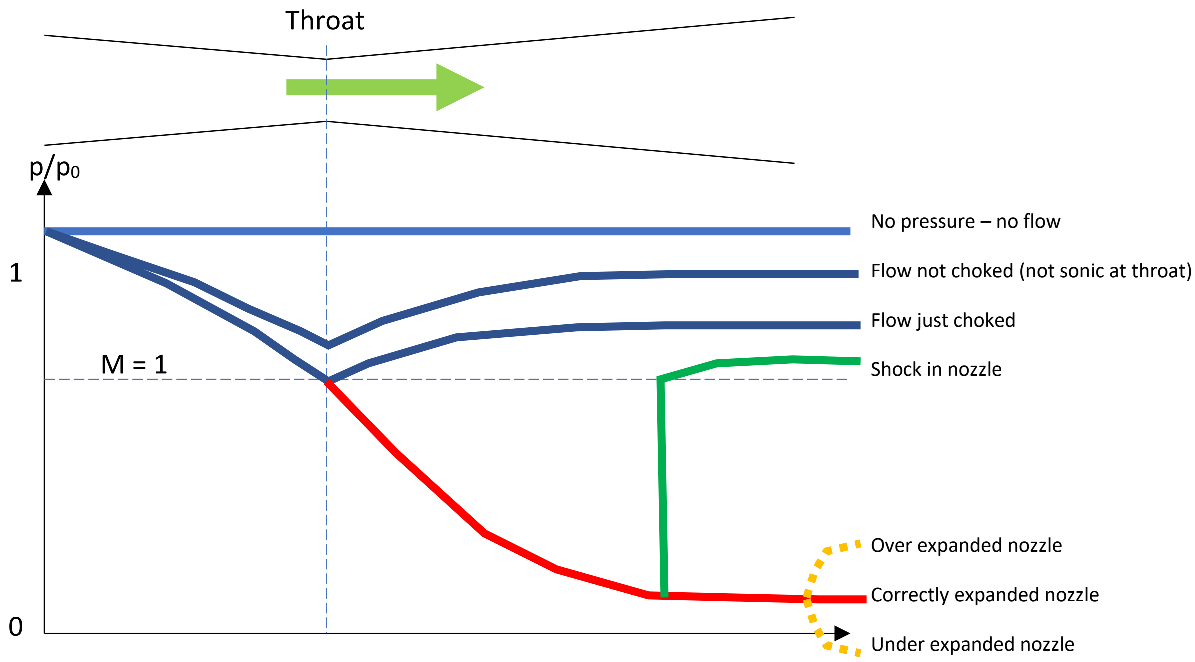
- Over-expanded (if $p_a > p_e$) occurs when the exit pressure is less than ambient.
- Under expanded (if $p_a < p_e$) occurs when the exit pressure is greater than ambient.

In both these cases expansion-fans and shockwaves are present in the flow as the pressures try to equalise (sometimes seen as shock-diamonds in the over-expanded exhaust of a jet). The general situation is shown in the diagram below:



(Image: “Rocket nozzle expansion” by Hohum. Origin: Wikipedia, license: listed as “copyright free, public domain” by author)

The pressure situations for all these circumstances are shown in the diagram below:



A good overall explanation is available at:

https://www.youtube.com/watch?v=0ycxMTUnruw

7. Conclusions

The discussion above examines the "classical" scramjet. However, there is one problem – it doesn't work! This is mainly because of the mixing and combustion issues outlined above in combination with the fine balance of thrust and drag and the difficulty of adding further heat to the already very hot air.

The difficulties have meant that alternatives and alterations have been researched to try and overcome the issues; for example, in our own work:

https://www.youtube.com/playlist?list=PLvv49eoNh3jSOmAYq0MWAAR3GCg50rK_h

Perhaps you will be the one to come up with the final solution!

Videos and playlists

University of Queensland Scramjet course:

https://www.youtube.com/channel/UCmaKEBNUuW8YcGx6miwwRwg

(click on playlists to see subject breakdown)

My scramjet videos:

https://www.youtube.com/playlist?list=PLvv49eoNh3jTckJA7Yy4zJPhu9S_PFbJH

Supplementary course notes

1. An introduction to compressible flow.
2. Shockwaves

References and reading

General references:

C. Segal, "The Scramjet Engine", Cambridge University Press, Cambridge, 2009.

W. H Heiser et al, "Hypersonic Airbreathing Propulsion", AIAA Education, Washington, 1994. *(presently available as an ebook through UHI library* https://www.uhi.ac.uk/en/libraries/*)*

Billig's scramjet model:

F.S. Billig, "Design and development of single stage to orbit vehicles, John-Hopkins Applied Physics Laboratory Technical Digest, 11 (3/4), pp.336-352, 1990.

Fluid Mechanics reference:

Notes on:

An Introduction to compressible flow

Shockwaves

J. Anderson, "Modern Compressible Flow", McGraw-Hill, 2012 (3rd ed) (main ref)

J. Anderson, "Fundamentals of Aerodynmaics", McGraw-Hill, 2001 (3rd ed)

Anderson's "Hypersonic and High Temperature Gas Dynamics" may also be useful.

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Notes version

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